

# CEEX Exponentiation in EQD

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The aim is to summarize briefly on:

- Main features of YFS/CEEX exponentiation in QED
- Examples of results relevant for LEP/LC physics program

Based on recent papers by:

S.J., D. Bardin, M. Melles, M. Skrzypek, W. Płaczek, E. Richter-Wąs, T. Riemann,  
B.F.L. Ward, and Z. Wąs:

Phys. Rev. D **63**, 113009 (2001),

Eur.Phys. J. **C24**,373, (2002),

Phys.Rev. **D65** 073030, (2002)

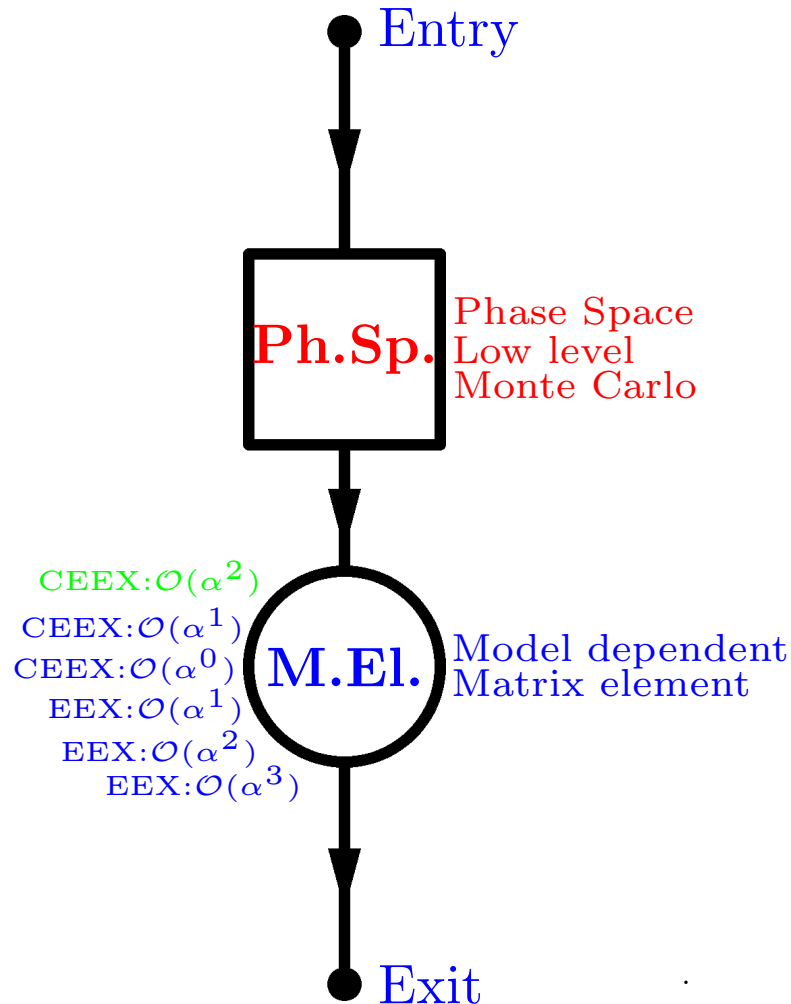
**These and related slides on <http://home.cern.ch/jadach>**

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### Standard Model calculations for LEP with YFS exponentiation

- $e^+e^- \rightarrow f\bar{f} + n\gamma$ ,  $f = \tau, \mu, d, u, s, c$ , YFS1 (1987-1989)  $\mathcal{O}(\alpha^1)_{exp}$  ISR,  
 YFS2  $\in$  KORALZ (1989-1990),  $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$  ISR  
 YFS3  $\in$  KORALZ (1990-1998),  $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$  ISR+FSR  
 KKMC (98-02)  $\mathcal{O}(\alpha^2 + h.o.LL)_{exp}$  ISR+FSR+Interf.  $d\sigma/\sigma = 0.2\%$
- $e^+e^- \rightarrow e^+e^- + n\gamma$  for  $\theta < 6^\circ$   
 BHLUMI 1.x, (1987-1990),  $\mathcal{O}(\alpha^1)_{exp}$   
 BHLUMI 2.x, (1990-1996),  $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$   $d\sigma/\sigma = 0.07\%$
- $e^+e^- \rightarrow e^+e^- + n\gamma$  for  $\theta > 6^\circ$   
 BHWIDE (1994-1998),  $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$
- $e^+e^- \rightarrow W^+W^- + n\gamma$ ,  $W^\pm \rightarrow f\bar{f}$   
 KORALW (1994-2001)
- $e^+e^- \rightarrow W^+W^- + n\gamma$ ,  $W^\pm \rightarrow f\bar{f}$   
 YFS3WW (1995-2001), YFS expon. + Leading Pole Approx.  $d\sigma/\sigma = 0.4\%$

Typical MC realization: “matrix element  $\times$  exact phase space” principle



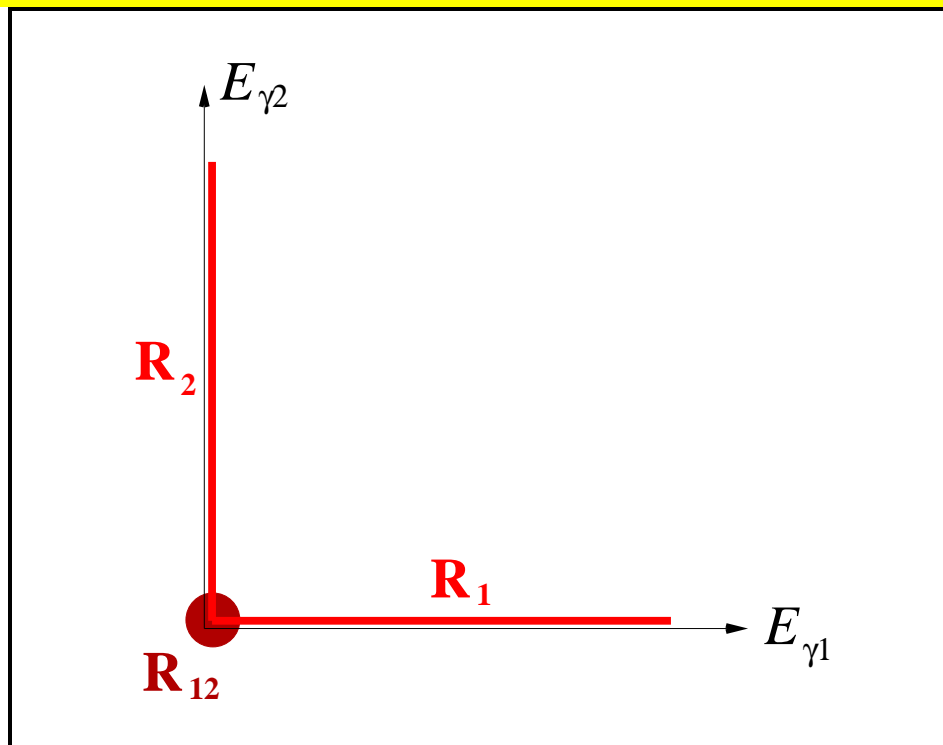
In the Monte Carlo realization it means:

- Universal exact Phase-space Monte Carlo simulator is a separate module producing “raw events” (with importance sampling)
- Library of several types of SM/QED matrix elements provides “model weight” is another independent module (KKMC example is shown)
- Tau decays and hadronization come after of course.

## Main steps in YFS exponentiation

- Reorganization of the perturbative complete  $\mathcal{O}(\alpha^\infty)$  series such that IR-finite  $\bar{\beta}$  components are isolated (factorization theorem).
- Truncate IR-finite  $\bar{\beta}$ s to finite  $\mathcal{O}(\alpha^n)$  and calculate them from Feynman diagrams recursively.

**Factorization for overlapping IR divergences for  $2\gamma$ :  $R_{12} \in R_1$  and  $R_{12} \in R_2$**



$$D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}); \quad p_{f_1} + p_{f_2} = p_{f_3} + p_{f_4}$$

$$D_1(p_f; k_1) = \bar{\beta}_0(p_f) \tilde{S}(k_1) + \bar{\beta}_1(p_f; k_1); \quad p_{f_1} + p_{f_2} \neq p_{f_3} + p_{f_4}$$

$$D_2(k_1, k_2) = \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1) + \bar{\beta}_2(k_1, k_2).$$

NOTE:  $\bar{\beta}_0$  and  $\bar{\beta}_1$  used beyond their usual (Born and  $1\gamma$ ) phase space.

A kind of smooth “extrapolation” or “projection” is always necessary.

**Recursive, order-by-order definition of IR-finite  $\bar{\beta}$ s; SUBTRACTIONS!**

$$D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}),$$

$$D_1(p_f; k_1) = \bar{\beta}_0(p_f) \tilde{S}(k_1) + \bar{\beta}_1(p_f; k_1)$$

$$D_2(k_1, k_2) = \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1) + \bar{\beta}_2(k_1, k_2).$$



$$\bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}),$$

$$\bar{\beta}_1(p_f; k_1) = D_1(p_f; k_1) - \bar{\beta}_0(p_f) \tilde{S}(k_1)$$

$$\bar{\beta}_2(k_1, k_2) = D_2(k_1, k_2) - \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) - \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1).$$

**In the practical order-by-order calculations**

**IR-finite  $\bar{\beta}$ s are truncated to certain fixed  $\mathcal{O}(\alpha^n)$**

**Classic EEX/YFS schematically;  $\beta$ 's truncated to  $\mathcal{O}(\alpha^1)$ , example of ISR**

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n)$$

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} e^{Y(m_\gamma)} D_n(q_1, q_2, k_1, \dots, k_n)$$

$$D_0 = \bar{\beta}_0$$

$$D_1(k_1) = \bar{\beta}_0 \tilde{S}(k_1) + \bar{\beta}_1(k_1)$$

$$D_2(k_1, k_2) = \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1)$$

$$D_n(k_1, k_2 \dots k_n) = \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2) \dots \tilde{S}(k_n) + \bar{\beta}_1(k_1) \tilde{S}(k_2) \tilde{S}(k_3) \dots \tilde{S}(k_n) \\ + \tilde{S}(k_1) \bar{\beta}_1(k_2) \tilde{S}(k_3) \dots \tilde{S}(k_n) + \dots + \tilde{S}(k_1) \tilde{S}(k_2) \tilde{S}(k_3) \dots \bar{\beta}_1(k_n)$$

**Real soft factors:**  $\tilde{S}(k) = \sum_{\sigma} |\mathfrak{s}_{\sigma}(k)|^2 = |\mathfrak{s}_+|^2(k) + |\mathfrak{s}_-|^2(k) = -\frac{\alpha}{\pi} \left( \frac{q_1}{kq_1} - \frac{q_2}{kq_2} \right)^2$

**IR-finite building blocks:**

$$\bar{\beta}_0 = \left( e^{-2\alpha\Re B_4} \sum_{\lambda} |\mathcal{M}_{\lambda}^{\text{Born+Virt.}}|^2 \right) \Big|_{\mathcal{O}(\alpha^1)}, \quad \lambda = \text{fermion helicities}, \quad \sigma = \text{photon hel.}$$

$$\bar{\beta}_1(k) = \sum_{\lambda\sigma} |\mathcal{M}_{\lambda\sigma}^{1-\text{PHOT}}|^2 - \sum_{\sigma} |\mathfrak{s}_{\sigma}(k)|^2 \sum_{\lambda} |\mathcal{M}_{\lambda}^{\text{Born}}|^2$$

**Everything in terms of  $\sum_{spin} |\dots|^2$  ! Distr.  $< 0$  possible for hard  $2\gamma$ .**

## New CEEEX replaces old EEX, both derived from YFS 1961

EEX= Exclusive EXponentiation, very close to original Yennie-Frautschi-Suura 1961

CEEEX = Coherent EXclusive exponentiation, is an extension of YFS

**COHERENT = Friendly to Quantum coherence**

Coherence among Feynman diags.

Complete  $\left| \sum_{diagr.}^n \mathcal{M}_i \right|^2$  rather than often incomplete  $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$ .

Narrow resonances,  $\gamma \oplus Z$  exchanges,  $t \oplus s$  channels, ISR $\oplus$ FSR, Angular ordering, etc.

**EEX: KORALZ/YFS2, BHLUMI, YFSWW, KORALZ**

**CEEEX: KKMC**

## CEEX schematically, the example of ISR $\mathcal{O}(\alpha^1)$

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n)$$

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda, \sigma_1, \dots, \sigma_n} |e^{\alpha B(m_\gamma)} \mathcal{M}_{n, \sigma_1, \dots, \sigma_n}^\lambda(k_1, \dots, k_n)|^2$$

$$\mathcal{M}_0^\lambda = \hat{\beta}_0^\lambda, \quad \lambda = \text{fermion helicities,}$$

$$\mathcal{M}_{1, \sigma_1}^\lambda(k_1) = \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}_{1, \sigma_1}^\lambda(k_1)$$

$$\mathcal{M}_{2, \sigma_1, \sigma_2}^\lambda(k_1, k_2) = \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) \mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}_{1, \sigma_1}^\lambda(k_1) \mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}_{1, \sigma_2}^\lambda(k_2) \mathfrak{s}_{\sigma_1}(k_1)$$

$$\begin{aligned} \mathcal{M}_{n, \sigma_1, \dots, \sigma_n}^\lambda(k_1, k_2, \dots, k_n) &= \hat{\beta}_0^\lambda \mathfrak{s}_{\sigma_1}(k_1) \mathfrak{s}_{\sigma_2}(k_2) \dots \mathfrak{s}_{\sigma_n}(k_n) + \hat{\beta}_{1, \sigma_1}^\lambda(k_1) \mathfrak{s}_{\sigma_2}(k_2) \dots \mathfrak{s}_{\sigma_n}(k_n) \\ &+ \mathfrak{s}_{\sigma_1}(k_1) \hat{\beta}_{1, \sigma_2}^\lambda(k_2) \dots \mathfrak{s}_{\sigma_n}(k_n) + \dots + \mathfrak{s}_{\sigma_1}(k_1) \mathfrak{s}_{\sigma_2}(k_2) \dots \hat{\beta}_{1, \sigma_2}^\lambda(k_2) \end{aligned}$$

**$\mathcal{O}(\alpha^1)$  IR-finite building blocks:**

$$\hat{\beta}_0^\lambda = \left( e^{-\alpha B_4} \mathcal{M}_\lambda^{\text{Born+Virt.}} \right) \Big|_{\mathcal{O}(\alpha^1)},$$

$$\hat{\beta}_{1, \sigma}^\lambda(k) = \mathcal{M}_{1, \sigma}^\lambda(k) - \hat{\beta}_0^\lambda \mathfrak{s}_\sigma(k)$$

**Everything done in terms of  $\mathcal{M}$ -amplitudes!**

**Distr.  $\geq 0$  by construction!**

**In KKMC the above is done up to  $\mathcal{O}(\alpha^2)$  for ISR and FSR**

## Full Scale CEEEX $\mathcal{O}(\alpha^r)$ , $r=1,2$ , master formula

### Polarized total x-section:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a + p_b; p_c, p_d, k_1, \dots, k_n) e^{2\alpha\Re B_4} \sum_{\sigma_i, \lambda, \bar{\lambda}} \sum_{i,j,l,m=0}^3$$

$$\hat{\mathcal{E}}_a^i \hat{\mathcal{E}}_b^j \sigma_{\lambda_a \bar{\lambda}_a}^i \sigma_{\lambda_b \bar{\lambda}_b}^j \mathfrak{M}_n^{(r)} \left( \begin{matrix} p & k_1 & k_2 & \dots & k_n \\ \lambda & \sigma_1 & \sigma_2 & \dots & \sigma_n \end{matrix} \right) \left[ \mathfrak{M}_n^{(r)} \left( \begin{matrix} p & k_1 & k_2 & \dots & k_n \\ \bar{\lambda} & \sigma_1 & \sigma_2 & \dots & \sigma_n \end{matrix} \right) \right]^* \sigma_{\bar{\lambda}_c \lambda_c}^l \sigma_{\bar{\lambda}_d \lambda_d}^m \hat{h}_c^l \hat{h}_c^m$$

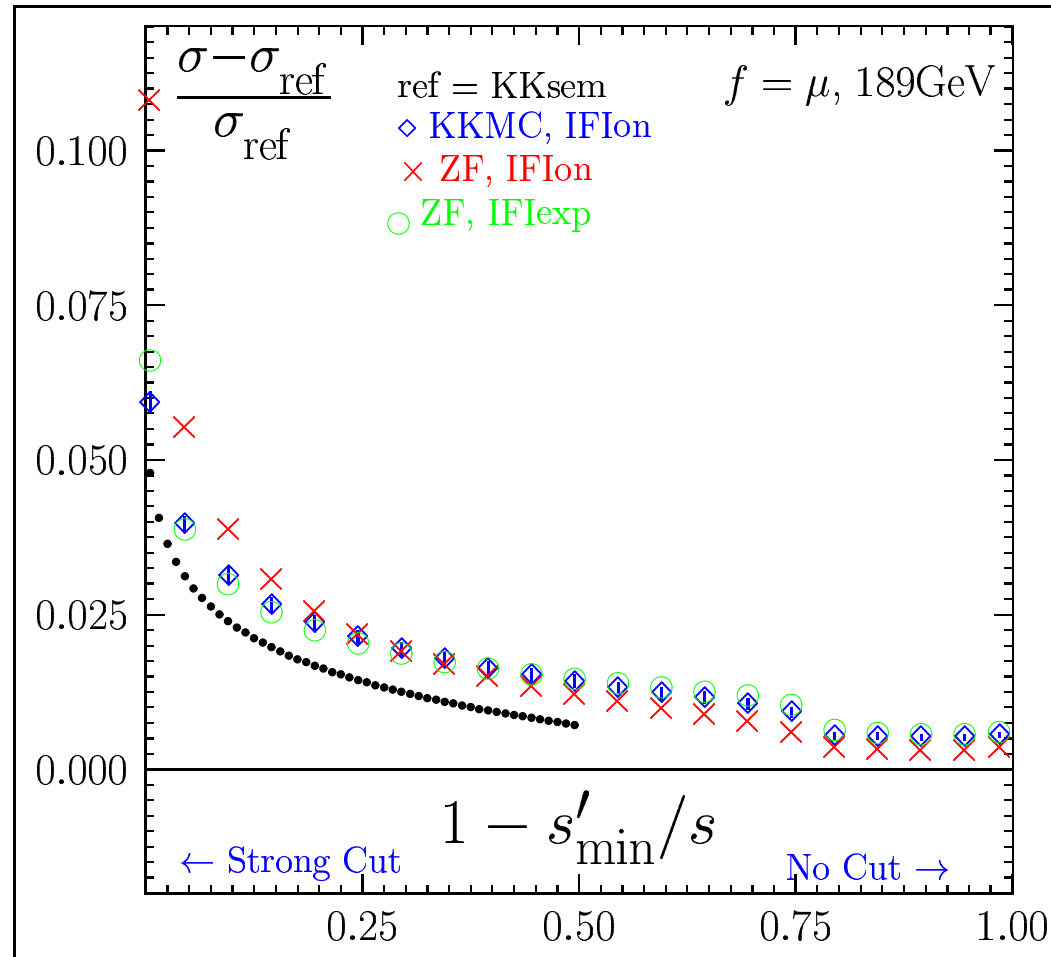
### CEEEX amplitudes:

$$\mathfrak{M}_n^{(1)} \left( \begin{matrix} p & k_1 & \dots & k_n \\ \lambda & \sigma_1 & \dots & \sigma_n \end{matrix} \right) = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^n \mathfrak{s}_{[i]}^{\{\wp_i\}} \left\{ \beta_0^{(1)} \left( \begin{matrix} p \\ \lambda \end{matrix}; X_{\wp} \right) + \sum_{j=1}^n \frac{\beta_{1\{\wp_j\}}^{(1)} \left( \begin{matrix} p & k_j \\ \lambda & \sigma_j \end{matrix}; X_{\wp} \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} \right\}$$

$$\mathfrak{M}_n^{(2)} \left( \begin{matrix} p & k_1 & \dots & k_n \\ \lambda & \sigma_1 & \dots & \sigma_n \end{matrix} \right) = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^n \mathfrak{s}_{[i]}^{\{\wp_i\}} \times \left\{ \beta_0^{(2)} \left( \begin{matrix} p \\ \lambda \end{matrix}; X_{\wp} \right) + \sum_{j=1}^n \frac{\beta_{2\{\wp_j\}}^{(1)} \left( \begin{matrix} p & k_j \\ \lambda & \sigma_j \end{matrix}; X_{\wp} \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} + \sum_{1 \leq j < l \leq n} \frac{\beta_{2\{\wp_j, \wp_l\}}^{(2)} \left( \begin{matrix} p & k_j & k_l \\ \lambda & \sigma_j & \sigma_l \end{matrix}; X_{\wp} \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}} \mathfrak{s}_{[l]}^{\{\wp_l\}}} \right\}$$

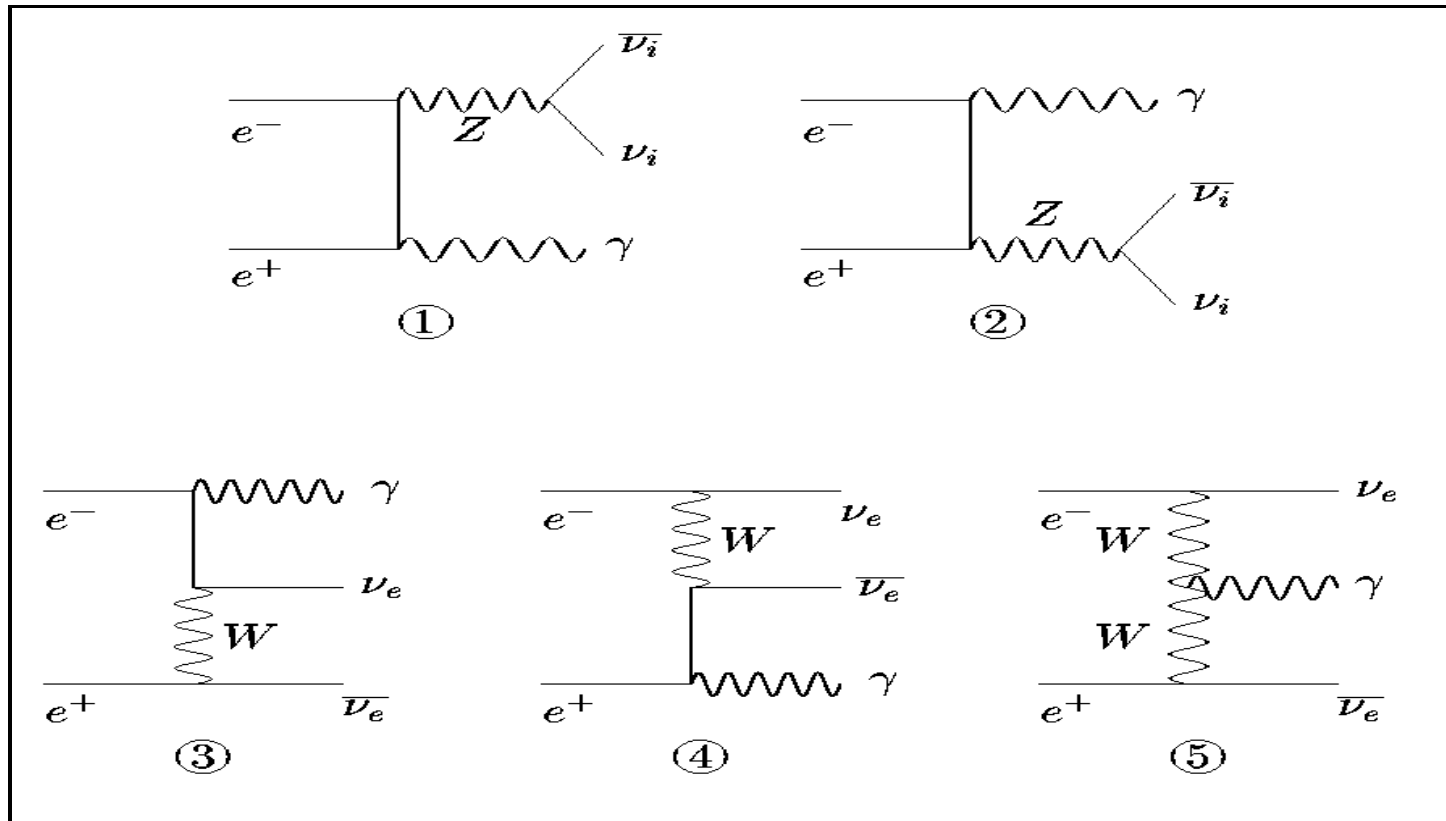
For details see Phys. Rev. D 63, 113009 (2001).

**KK MC vs. Zfitter for ISR+FSR, including IFI=ISR $\otimes$ FSR;  $d\sigma/\sigma = 0.2\%$**



**Comparison of KKMC and Zfitter. ISR of Zfitter is based on the  $\mathcal{O}(\alpha^2)$  result by Berends, Burgers, VanNeerven (1984), while **KKMC is totally independent!****

## Improvements of CEEEX in KKMC for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ process

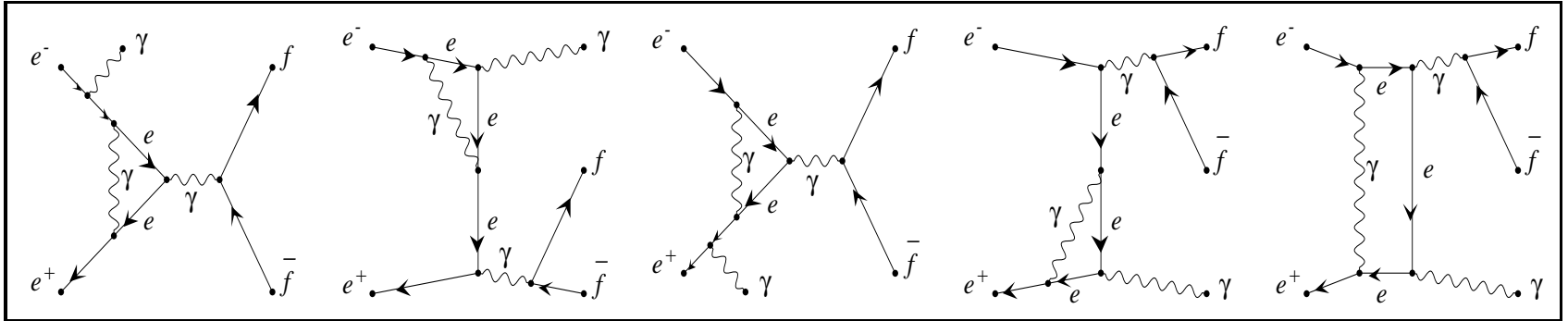


- $KK$  MC with CEEEX matrix element is now extended to the neutrino mode. (Eur.Phys.J.C24:373-383,2002, hep-ph/0110371)
- It is a replacement for KORALZ older program.
- Useful for LEP and 1-st step towards LC.

**Improvements of CEEX in KKMC for  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  process**

- The systematic error is estimated to be 1.3% for  $\nu_e\bar{\nu}_e\gamma$  and 0.8% for  $\nu_\mu\bar{\nu}_\mu\gamma$  and  $\nu_\tau\bar{\nu}_\tau\gamma$ .
- For observables with two observed photons we estimate the uncertainty to be about 5%.
- These new improved results were obtained thanks to the inclusion of non-photonic electroweak corrections of the ZFITTER package and due to newly constructed, exact, single and double emission photon amplitudes in the  $\mathcal{K}\mathcal{K}$  MC for the contribution with the  $t$ -channel  $W$  exchange.
- The virtual corrections for the  $W$  exchange are at present introduced in the approximated form. The exponentiation scheme CEEX is the same as in the original  $\mathcal{K}\mathcal{K}$  MC program

## Exact Differential $\mathcal{O}(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^- \rightarrow 2f$

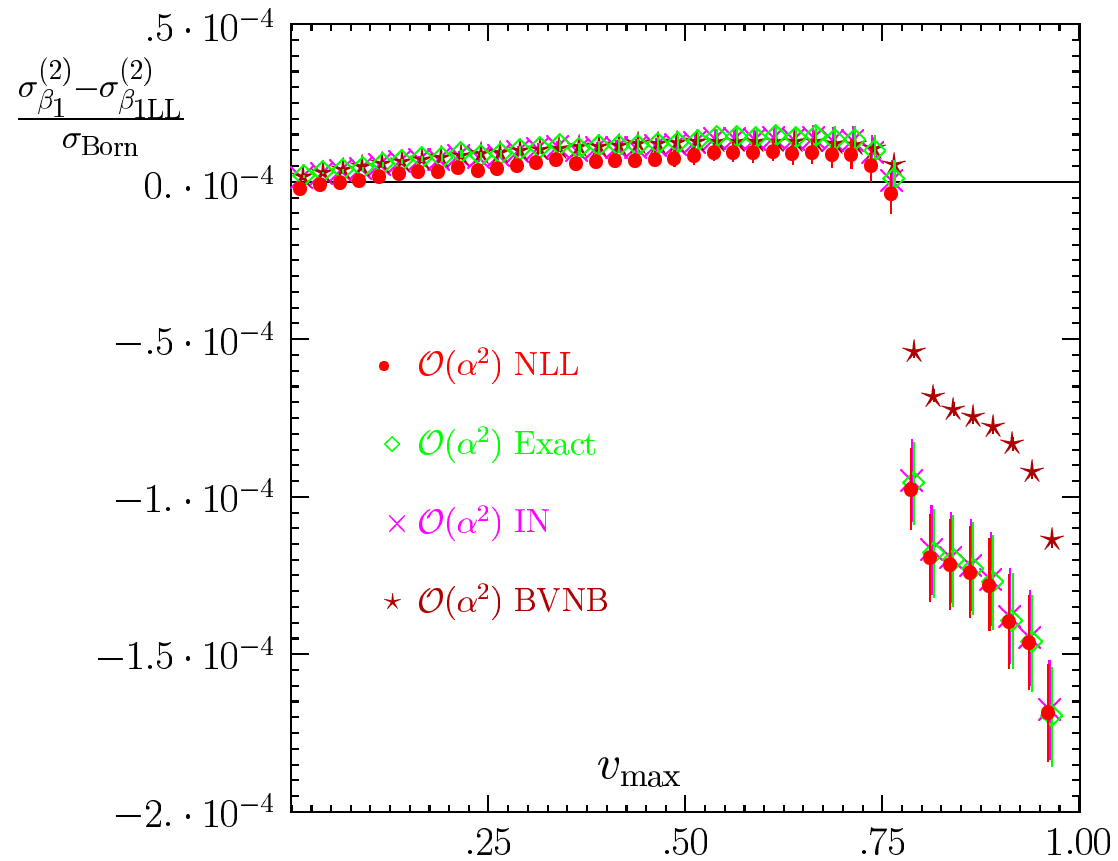


Phys.Rev.D65:073030,2002; hep-ph/0109279

- Fully differential results for  $2f + 1\gamma_{virt} + 1\gamma_{real}$  checked against Igarashi and Nakazawa
- Partly integrated results checked against Berends, Burgers and VanNeerven
- Important component for any exact  $\mathcal{O}(\alpha^2)$  exponentiated calculation for  $e^+e^- \rightarrow 2f$  is now available.

Similar works were also completed by Karlsruhe group recently, hep-ph/0204283 .

## Exact Differential $\mathcal{O}(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^- \rightarrow 2f$



**For  $v < 0.9$  the agreement to within  $0.5 \cdot 10^{-4}$  reached**

**Conclusions**

- YFS inspired EEX and CEEX schemes are successful examples of the Monte Carlo based directly on the factorization theorem (albeit for IR soft case for abelian QED only).
- Work well in practice: KORALZ, BHLUMI, YWSWW3, BHWIDE, KK MC.
- Extension (as far as possible) to all collinear singularities would be very desirable and practically important!
- The KKMC program is extended to neutrino channel
- Missing fully differential  $2f + 1\gamma_{virt} + 1\gamma_{real}$  distributions for  $\mathcal{O}(\alpha^2)$  CEEX are now available