Theoretical Errors for WW Observables

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IN COLLABORATION WITH

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- **Introduction; How to quantify TH errors?**
- **TH errors in total cross section**
- **TH errors in M_W**
- **Differential distributions**
- **Summary**

Motto:

What is the theoretical error?
It is “what we do not know” measured in permile units.

The Art (Science?) of Estimating TH Errors.

[a] Complete (direct) evaluation

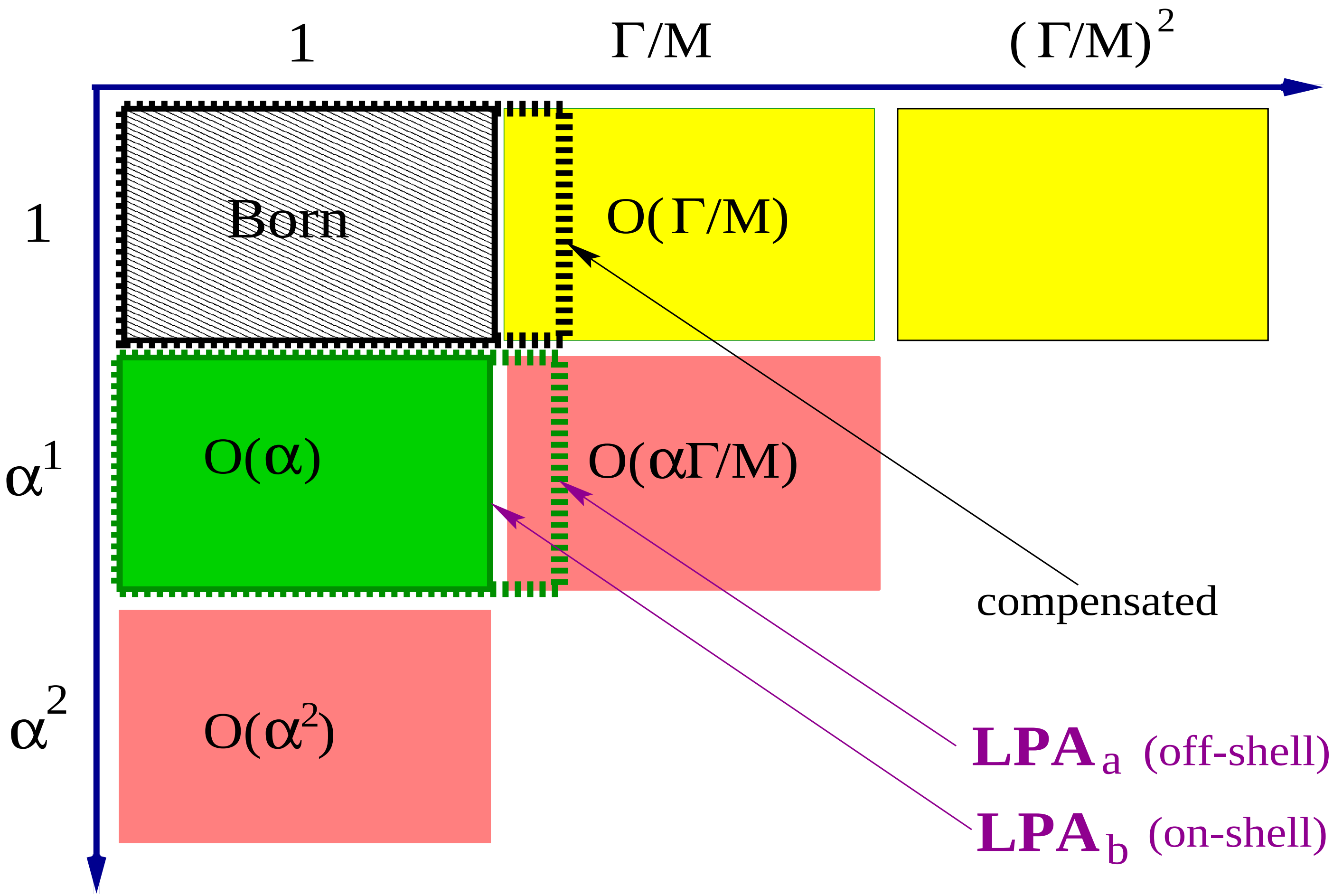
[b] Incomplete (indirect) evaluation:

[b1] Variation of the perturbative scheme (exponentiation, LPA, LLogs)

[b2] Parametric method: vary parameters in the the calculation (program)

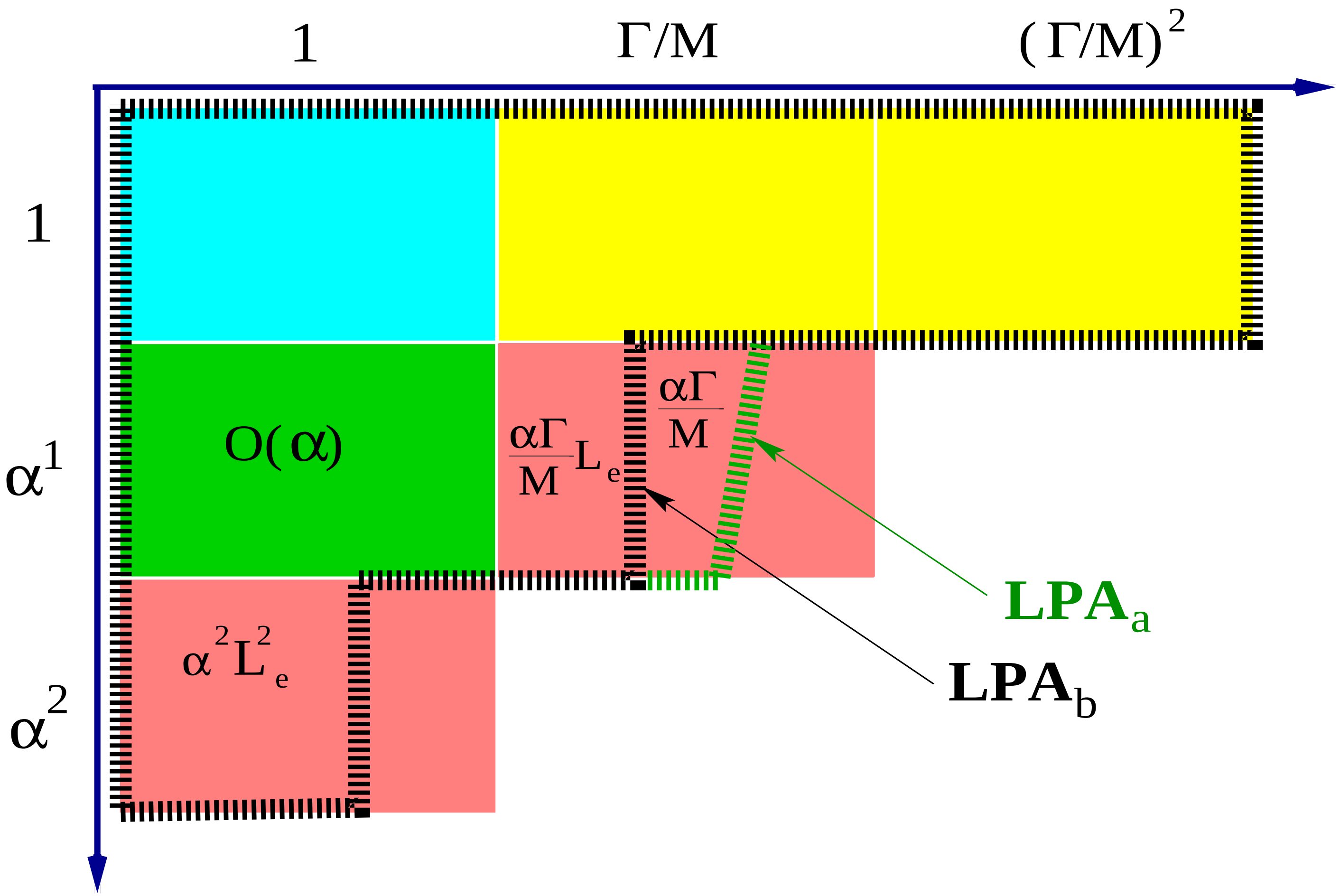
[c] Scale parameter $\times$ safety factor;
NB. safety factor may be from numerical exercise of the type [b]

Double expansion in α and Γ_W/M_W in $e^+e^- \to 4f$



The difference between two LPA schemes (R. Stuart 1996) is generally $\mathcal{O}(\Gamma_W/M_W) \sim 2\%$ at the tree level and $\mathcal{O}(\alpha\Gamma_W/M_W) \sim 10^{-4}$ at $\mathcal{O}(\alpha^1)$.

What is included in YFSWW3&KoralW ?



The physics content of RacoonWW is quite similar.

TH errors for σ_{tot}

$\Delta\sigma/\sigma$				
Error Type	Scale Param.	Recipe	number	Err. estim.
ISR LL	$(\frac{\alpha}{\pi})^4 L_e^4 = 2\cdot 10^{-5}$	$\frac{1}{\sigma}(\sigma_{KW}(\alpha^3 L_e^3) - \sigma_{KW}(\alpha^2 L_e^2))$	$\leq 2\cdot 10^{-4}$	$\leq 2\cdot 10^{-4}$
ISR NLL	$(\frac{\alpha}{\pi})^2 L_e = 1.7\cdot 10^{-4}$	$\frac{1}{\sigma}\frac{1}{L_e}(\sigma_{KW}(\alpha^2 L_e^2) - \sigma_{KW}(\alpha^1 L_e^1))$	$\sim 2\cdot 10^{-4}$	$\sim 2\cdot 10^{-4}$
Next-to-LPA	$\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W} = 0.7\cdot 10^{-4}$	$\mathrm{LPA}_a - \mathrm{LPA}_b$ of YFSWW3	$\sim 1\cdot 10^{-3}$	$\sim 1\cdot 10^{-3}$
Non-Factorizable	$\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W} = 0.7\cdot 10^{-4}$	RacoonWW − YFSWW3	$\sim 3\cdot 10^{-3}$	$\sim 1\cdot 10^{-3}$
		Coulomb screened−unscr.	$\sim 1\cdot 10^{-3}$	$\sim 1\cdot 10^{-3}$
FSR (W decays)	$(\frac{\alpha}{\pi})\ \epsilon = 5\cdot 10^{-4};\ \epsilon \simeq 0.2$	PHOTOS versus $\mathcal{O}(\alpha)$ M.C.	$\sim 1\cdot 10^{-3}$	$\sim 1\cdot 10^{-3}$
light pairs $\vert_{real}^{virt.}$	$(\frac{\alpha}{\pi})^2 L_e^2 = 4\cdot 10^{-3}$	KorWan	$?.?\cdot 10^{-3}$	$?.?\cdot 10^{-3}$
$\mathcal{O}(\alpha^2)_{\mathrm{EW}}^{\mathrm{Non-ISR}}$	α^2	$\frac{1}{2}\delta_{\mathrm{NL}}^2$ YFSWW3	$2\cdot 10^{-4}$	$4\cdot 10^{-4}$
????	$?\times\alpha^?$	???	$?\cdot 10^{-?}$	$?\cdot 10^{-?}$

$L_e = \ln\frac{s}{m_e^2} \simeq 25$, σ_{KW} from KoralW.

YFSWW3 lacks spin correlations in $\mathcal{O}(\alpha)$ part, $\sim 10^{-3}$ effect. Comparison with RacoonWW confirms that.

Strictly forbidden to combine the above errors!

Remarks on energy dependence: see next slide.

Energy dependence of TH errors for σ_{tot}

Close to the threshold LPA=DPA the prediction of YFSWW3$\&$KoralW should be interpreted as $Born_{4f} \otimes ISR$, (i.e. convolution of four-fermion Born with ISR), taking into account Coulomb correction.

This is sufficient to claim TH precision 2% towards the threshold.

YFSWW3-1.14 ↔ RacoonWW @ LEP2 Energies

A. Denner, S. Dittmaier, M. Roth, D. Wackeroth

$\sqrt{s}\,[GeV]$	$\sigma_{WW}\,[pb]$		(Y − R)/Y [%]
	YFSWW3	RacoonWW	
168.000	9.8302 (34)	9.8392 (49)	−0.09 (6)
172.086	12.0988 (41)	12.0896 (76)	0.08 (7)
176.000	13.6360 (45)	13.6271 (66)	0.07 (6)
180.000	14.7791 (49)	14.7585 (72)	0.14 (6)
182.655	15.3610 (50)	15.3684 (76)	−0.05 (6)
185.000	15.7755 (48)	15.7716 (78)	0.25 (6)
188.628	16.2664 (53)	16.2486 (111)	0.11 (8)
191.583	16.5680 (57)	16.5188 (85)	0.30 (6)
195.519	16.8409 (61)	16.8009 (87)	0.24 (6)
199.516	17.0167 (68)	16.9791 (88)	0.22 (6)
201.624	17.0755 (62)	17.0316 (89)	0.26 (6)
205.000	17.1279 (55)	17.0792 (89)	0.28 (6)
208.000	17.1507 (67)	17.0942 (90)	0.33 (7)
210.000	17.1467 (66)	17.0858 (91)	0.34 (7)
215.000	17.0786 (70)	17.0378 (91)	0.24 (7)

Agreement Within 0.4%

TH errors for M_W

- **ISR and other WW-production related corrections:**

  $\delta M_W \sim \Gamma_W \frac{\Gamma_W}{M_W} \times \varepsilon,$ (<1MeV),

  where $\varepsilon \simeq (\frac{\alpha}{\pi})^2 L_e,\;\; \varepsilon \simeq (\frac{\alpha}{\pi})^4 L_e^4, \varepsilon \simeq \alpha^2$,... etc.

- **FSR in W decays related corrections:**

  $\delta M_W \sim \Gamma_W \times \varepsilon^2$ (<5MeV ?),

  where $\varepsilon \simeq q_f^2 (\frac{\alpha}{\pi}) \ln(\frac{\Gamma_W}{M_W})(-2) \ln(\theta_{CALO})$ for CALO

  or $\varepsilon \simeq q_f^2 (\frac{\alpha}{\pi}) \ln(\frac{\Gamma_W}{M_W}) \ln(\frac{M_W^2}{m_f^2})$ for BARE.

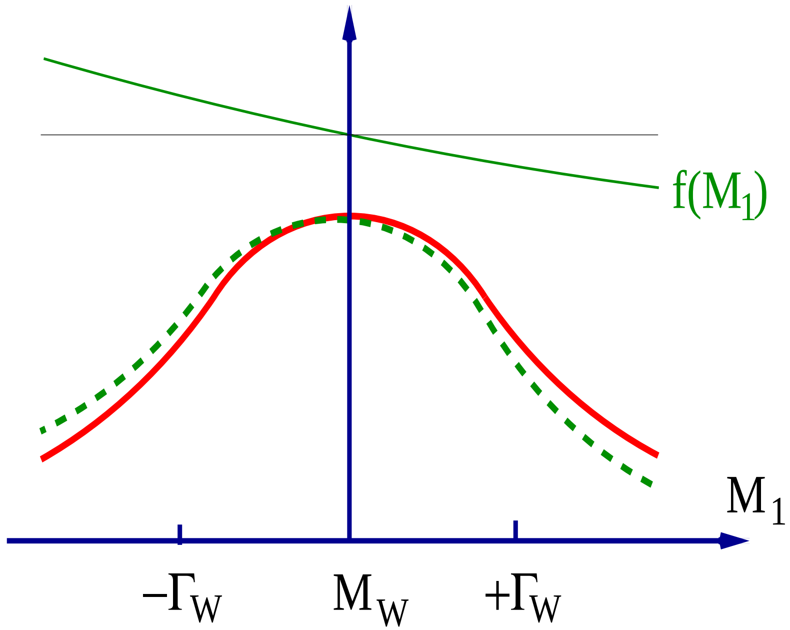
- **Non-factorizable ISR*FSR, Coulomb:**

  $\delta M_W \sim \Gamma_W \times \varepsilon^2$ (<1MeV),

  where $\varepsilon \simeq q_e q_f \frac{\alpha}{\pi}$

See next slides for “derivations”.

The influence of the corrs. from WW production (IRS etc.) on the W mass



$\rho(M_1) = \frac{d\sigma}{dM_1} \simeq |BW(M_1)|^2 \times f(M_1/\sqrt{s})$,

where f_1 is mild function of $x = M_1/\sqrt{s}$ related to WW production (ISR etc.)

$\delta M_W \sim \Gamma_W \frac{f(x_W + \frac{\Gamma_W}{M_W}) - f(x_W - \frac{\Gamma_W}{M_W})}{f(x_W)} \simeq \Gamma_W \frac{\Gamma_W}{M_W} \frac{f'(x_W)}{f(x_W)} \simeq \Gamma_W \frac{\Gamma_W}{M_W} \mathcal{O}(1)$.

What is response of δM_W due to $f(x) \to f(x) + \varepsilon f_1(x)$? Here εf_1 is H.O. corr.

$\Delta\delta M_W \sim \Gamma_W \frac{\Gamma_W}{M_W} \varepsilon \frac{f_1'(x_W)}{f(x_W)} \sim \Gamma_W \frac{\Gamma_W}{M_W} \varepsilon \times \mathcal{O}(1)$

Missing NLL $\mathcal{O}(\alpha^2)$ ISR gives $\varepsilon \sim \alpha^2 L_e$; 2γ from W's, $\varepsilon \sim \alpha^2$, etc. etc.

The influence of FSR in W decays on the W mass

$\frac{d\sigma}{dM_1^2}(M_1^2) \simeq \int dz\, \gamma_{FSR}(1-z)^{\gamma_{FSR}-1} \frac{d\sigma}{dM_1^2}(zM_1^2)$,

where $\gamma_{FSR} =\simeq q_f^2(\frac{\alpha}{\pi}) \ln(\frac{M_W^2}{m_f^2})$ for BARE

and $\gamma_{FSR} =\simeq q_f^2(\frac{\alpha}{\pi}) 2\ln(\frac{M_W}{p^T_{CALO}}) \simeq 0.007$ for CALO.

Mass shift $\delta M_W \sim \Gamma_W \varepsilon;\ \ \varepsilon \sim \gamma_{FSR} \ln(\frac{M_W}{\Gamma_W}) \simeq 0.02$

is accounted for in $\mathcal{O}(\alpha)$–complete calculation.

The missing $\mathcal{O}(\alpha^2)$ effect can be: $\Delta\delta M_W \sim \Gamma_W \frac{1}{2}\varepsilon^2 < 1$MeV.

The influence N-F QED interferences on the W mass

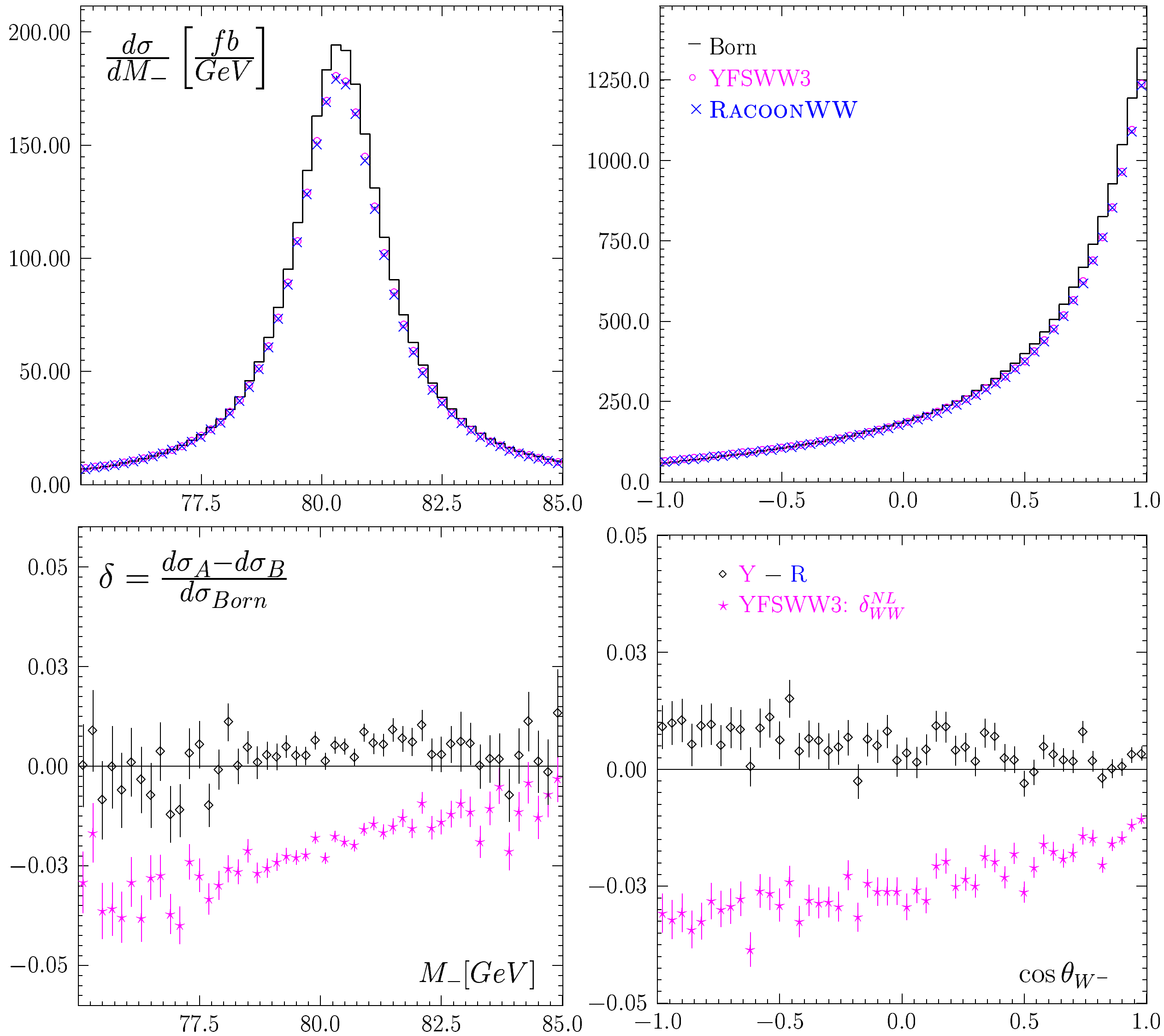
$\frac{d\sigma}{dM_1^2}(M_1^2) \simeq |BW(M_1)|^2 \cdot \frac{\alpha}{\pi} F(\frac{M_1-M_W}{\Gamma_W})$

$\delta M_W \sim \Gamma_W q_e q_f \frac{\alpha}{\pi} \sim 5$MeV is accounted for (screened Coulomb),

and $\Delta\delta M_W \sim \Gamma_W (\frac{\alpha}{\pi})^2 < 1$MeV can be missing.

YFSWW3 $\leftrightarrow$ RacoonWW

$$e^+e^- \longrightarrow W^+W^- \longrightarrow u\bar{d}\mu^-\bar{\nu}_\mu$$



$M_{W^-}^{inv}$, $\cos\theta_{W^-}$ @ $\sqrt{s} = 200$ GeV

Differential distributions

Proposal for an “error band”:

“Having a closer look into the results presented for the LEP2 MC workshop, we read off the following band for the differences:

W production angle: $0.5\% * \min[2 - \cos(\theta_W), 2]$

W invariant mass: $1\% * \max[1, |M - M_W|/\Gamma_W]$

This estimate results from the comparison at 200GeV, but we think it could be taken for the entire LEP2 range above 170GeV.”

(email from Stefan.)

Differential distributions

$d\sigma/d\cos\theta_W$:

The “error band” for TH error is probably not enough.

It reflects mainly the overall normalization error of σ_{tot}.

For TGC analysis one should also estimate TH error (?) for the slope,

(parametric method, R-Y comparisons etc.),

for example using the average $\langle(1-\cos\theta_W)^\beta\rangle$, $\beta = 1, 2, ?$.

$d\sigma/d\theta_\gamma$:

This is a separate subject.

$d\sigma/dE_\gamma$:

This is a separate subject.

Summary

- **The comparison YFSWW3&KoralW versus RacoonWW remains a cornerstone of the TH error estimate for WW observables.**
- **We stress on the richness of the structure and the origin of the TH errors, pointing to many possible methods of quantifying them.**
- **The generic estimate of 0.4% error for σ_{WW} remains as solid as before.**