Ununtegrated NLO evolution kernel for Monte Carlo modelling of the QCD/QED initial state radiadion KRKMC Project

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Most Desirable Monte Carlo for Colliders

What is the most desirable and difficult type of Monte Carlo event generator for QCD/QED/EW/BSM for data analysis at LHC, ILC and other colliders?

The MC implementing the Resummed and Fixed-Order perturbative Matrix Element with complete NLO in both!

NLO Parton Shower MC $\otimes$ NLO ME for hard prosess

What is missing?

Parton Shower Monte Carlo featuring

complete NLO (collinear) Evolution

does not exists yet!

WHY? The existing collinear factorization theorems in QCD/QED not very well suited for the Monte Carlo. No need of precise predictions from QCD in pre-LHC era, and more...

The name of the game

Can we construct Parton Shower Monte Carlo for QCD/QED Initial State Radiation:

- based firmly on Feynman Diagrams (ME) and LIPS,
- based rigorously on the collinear factorization (EGMPR, CSS),
- implementing exactly NLO $\overline{MS}$ DGLAP evolution **?**
- implementing fully unintegrated PDFs (FunPDF);
  NLO evolution done by MC itself using EXClusive NLO kernels.

We are going to show that YES, we can!

And to show first numerical implementation – proof of the concept.

Why bother? Potential gains:

PS MC implementing DGLAP NLO evolution on its own (standard PDFs not used) has huge potential of further improvements:

- **Heavy mass thresholds and low Q^2 limit**
- **Better low x limit: BFKL, CCFM**
- **Better link between resummed ME in parton shower and fixed-order ME of the hard process:**
  positive MC weights, no artificial cut at $k^T = 20$GeV.
- **easier to correct soft limit (colour), and more...!**

All the above very desirable for the "precision MC" projects for LHC and other colliders.

So why not done yet? (a) prohibitively difficult, (b) no need of high precision in the past colliders, (c) lack of fast CPUs.

Time evolution of the Collinear QCD Evolution

1970 1980 1990 2000 2010

LO

Moments OPE: (74) QCD: Georgi+Politzer

Diagramatic: (72) QED: Gribov+Lipatov; (77) QCD: Altarelli+Parisi

Monte Carlo: 10 years → (85 Sjostrand) (88) Webber

NLO

WE ARE HERE!!!

Moments OPE: (78) Floratos+Ross+Sachrajda

Diagramatic: (81) Curci+Furmanski+Petronzio

Monte Carlo: 27 years → (08) KRKMC

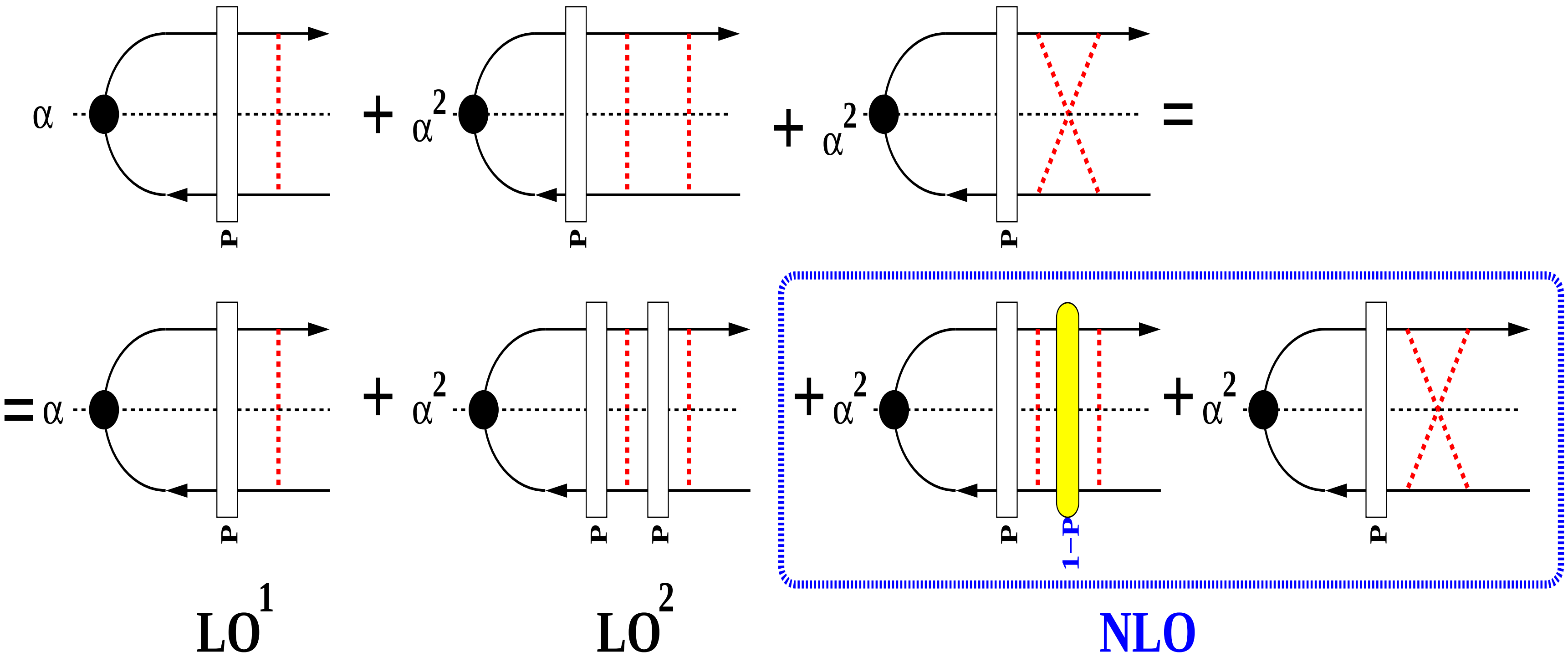
NNLO

Moments: (03) Moch+Verm.+Vogt

Diagramatic

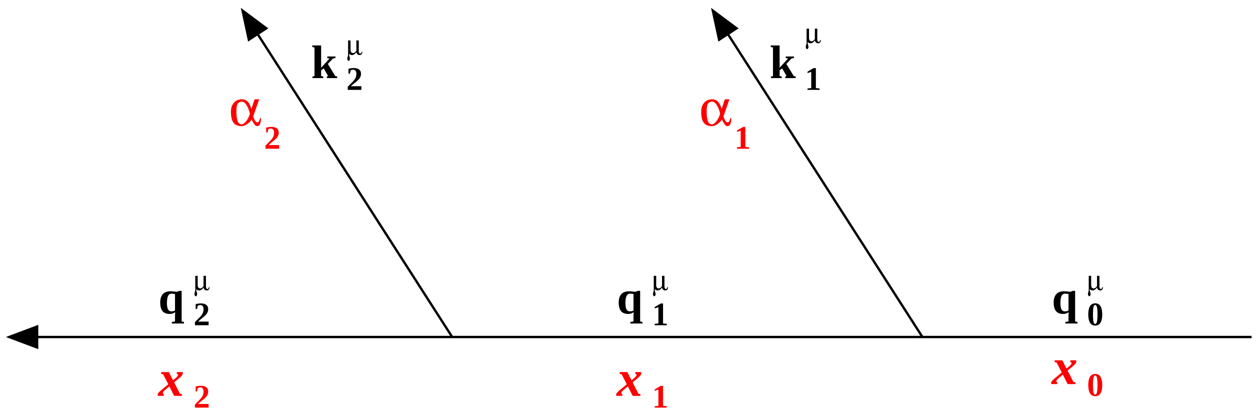
Monte Carlo: (15) ???

CFP: Extraction of NLO kernel from Feynman diags.



- CFP = Cuci-Furmanski-Petronzio (1980-82)
- Double gluonstrahlung, only 2 diagrams, $C_F C_A$ part only.
- Axial gauge, projection operator $P = P_{spin} P_{kin.} P_{pole}$.
- LO: $\mathcal{P}(z) = \frac{1+z^2}{2(1-z)}$
- NLO: $\mathcal{P}^N(z) = \frac{1+3x^2}{16(1-x)} \ln^2(x) + \frac{2-x}{4} \ln(x) + \frac{3}{8}(1-x)$.

Kinematics of double gluonstrahlung



- Initial parton $q_0^\mu = (E, 0, 0, E)$
- Emitted gluons: $k_i^\mu = (k_i^0, \mathbf{k}_i, k_i^3)$, $k_{Ti} = |\mathbf{k}_i|$.
- Lightcone plus variables: $x_i = \frac{q_i \cdot \zeta}{q_0 \cdot \zeta}$, $\alpha_i = \frac{k_i \cdot \zeta}{q_0 \cdot \zeta}$, $\zeta = (1, 0, 0, -1)$.

Unintegrated (exclusive) NLO kernel

$$\alpha'^2 \mathcal{P}^N(x) = \frac{1}{2!}\int \frac{d^3k_2}{2k_2^0}\int \frac{d^3k_1}{2k_1^0}\;\delta_{1=\max(|\mathbf{k}_1|,|\mathbf{k}_2|)/Q}\;\delta_{1-x=\alpha_1+\alpha_2}\; b_2^N(k_1,k_2)$$

where the UNINTEGRATED exclusive NLO kernel is

$$b_2^N(k_1,k_2) = \frac{(\alpha')^2}{16(2\pi)^2}\left[b^{Ladd.}(k_1,k_2) + b^{Ladd.}(k_2,k_1) + b^{XLad.}(k_1,k_2) - b^{Count.}(k_1,k_2)\right],$$

$$b^{Ladd.}(k_1,k_2) = \frac{1}{q^4(k_1,k_2)}\left\{\frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,0)}{\alpha_2^2}\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2}\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2}\right\},$$

$$b^{Xlad.}(k_1,k_2) = \frac{1}{q^4(k_1,k_2)}$$

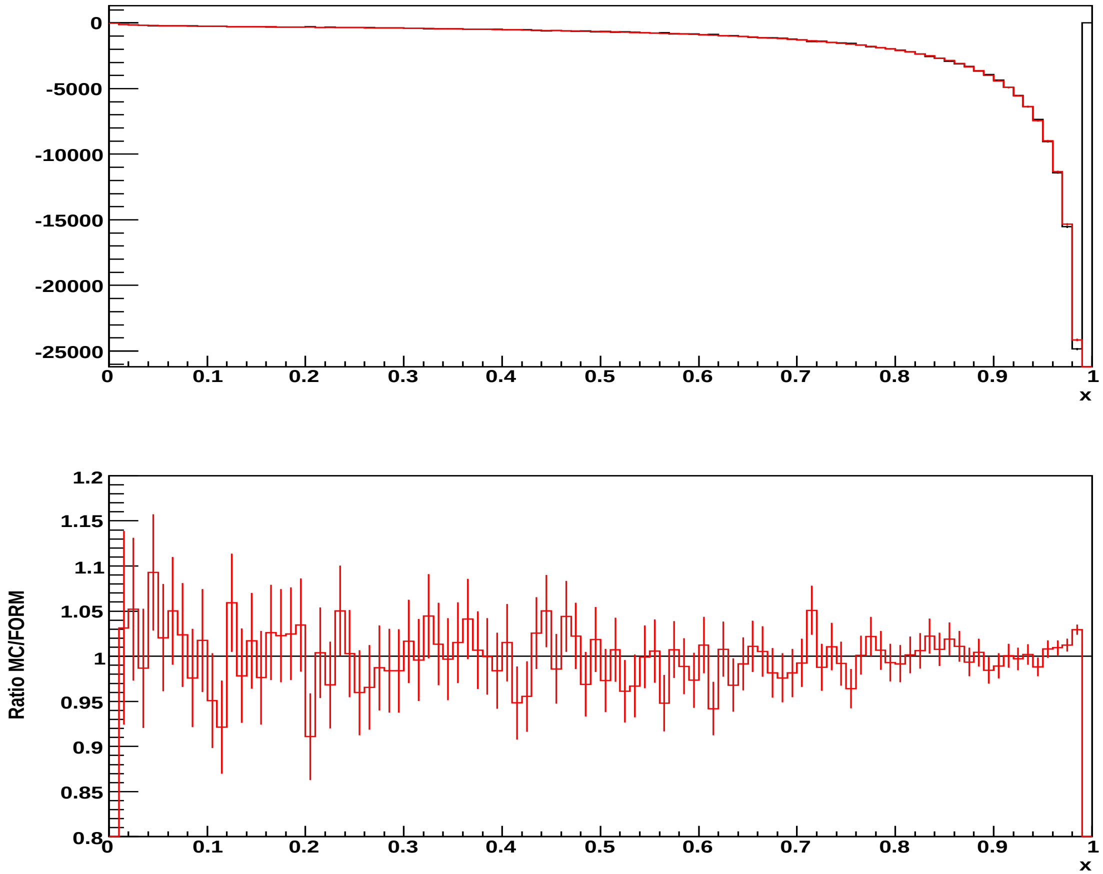
$$\times\left\{\frac{2T_1^x(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} - T_{2a}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_1\mathbf{k}_2^2} - T_{2b}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_2\mathbf{k}_1^2} + T_3^x(\alpha_1,\alpha_2)\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{\mathbf{k}_1^2\mathbf{k}_2^2}\right\},$$

$$b^{Count.}(k_1,k_2) = \frac{\mathbf{k}_2^2}{q^4(0,k_2)}\,\frac{T_2(\alpha_1,\alpha_2,0)}{\alpha_2^2}\,\frac{1}{\mathbf{k}_1^2} = \frac{T_2(\alpha_1,\alpha_2,0)}{(1-\alpha_1)^2\mathbf{k}_1^2\mathbf{k}_2^2}.$$

$$-q^2(k_1,k_2) = \frac{1-\alpha_2}{\alpha_1}\,\mathbf{k}_1^2 + \frac{1-\alpha_1}{\alpha_2}\,\mathbf{k}_2^2 + 2\mathbf{k}_1\cdot\mathbf{k}_2.$$

For simplicity we omitted terms due to ϵ part in in the γ-trace T^2.

Numerical cross-check on unintegrated kernel using FOAM

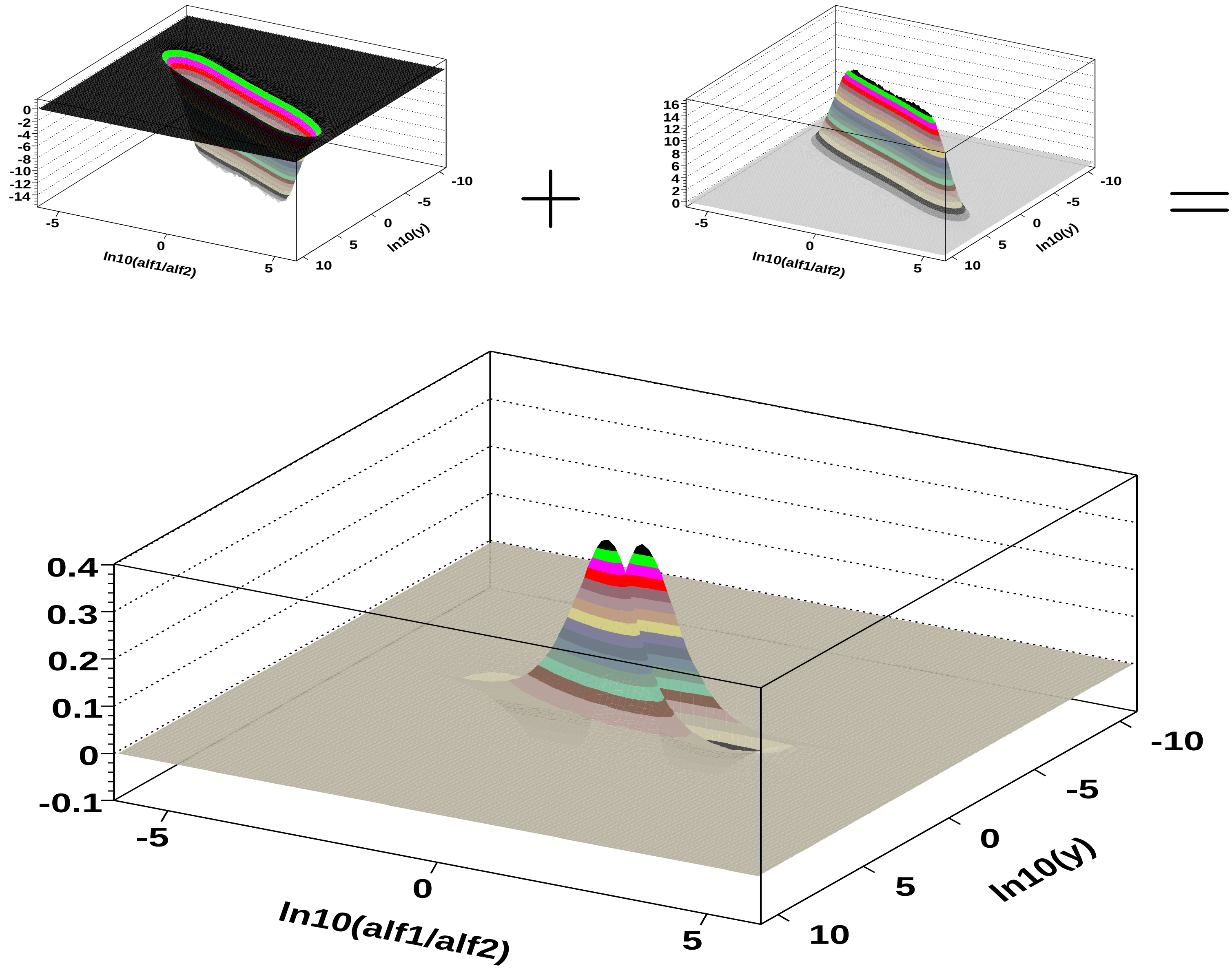


We are checking here whether

$$\alpha'^2 \mathcal{P}^N(z) = \frac{1}{2!} \int \frac{d^3 k_2}{2k_2^0} \int \frac{d^3 k_1}{2k_1^0}\ \delta_{1=\max(|\mathbf{k}_1|,|\mathbf{k}_2|)/Q}\ \delta_{1-x=\alpha_1+\alpha_2}\ b_2^N(k_1, k_2)$$

$$= \alpha'^2 \left(\frac{1+3x^2}{16(1-x)} \ln^2(x) + \frac{2-x}{4} \ln(x) + \frac{3}{8}(1-x) \right)$$

Unintegr. NLO kernel on the $(\ln k_{Ti},\ \ln \alpha_i)$ plane



$1 - x = \alpha_1 + \alpha_2$, $y = \frac{k_{T1}}{k_{T2}}$. Bose-symmetrized. **Short-range correlat.!**

Unintegrated NLO kernel: DISCUSSION

- Bose symmetrization removes double-log cancellations between distant regions of the phase space
- Adding interference graph removes soft IR singularity
- Nonzero when $kT1 \sim kT2$ or $\alpha_i \sim 1$;
  Just short range correlation disappearing in the soft limit!
- The same pattern when using virtuality as the evolution time variable (done)
- What next? Reinstallation of the NLO unitegrated kernel in the LO Monte Carlo, by means of reweighting LO events.

Implanting NLO UNintegrated kernel into parton shower MC

THE FRAMEWORK:

- **Markovian MC with standard integrated LO+NLO DGLAP kernel (gluonstrahlung part)**
- **Markovian MC with UNINTEGRATED LO+NLO DGLAP kernel. NEW!!!**
- **Analytical integration leading to NLO DGLAP kernel following Curci-Furmanski-Petronzio (1980); Feynman diagrams $\times$ LIPS, see HERA-LHC, May 08.**

Underlying simple Markovian LO parton shower MC

- Transverse momentum $q = e^t = k^T$ as evolution variable
- Mapping of q_i, x_i and φ_i into 4-momenta k_i^μ in the standard LIPS:
  $k_j^T = e^{t_i},\ k_j^- = 2(x_{j-1} - x_j)E_h,\ k_j^+ = (k_j^T)^2/k_j^-.$
- LO DGLAP evol. kernel $\mathcal{P}^L(z)$ for pure gluonstrahlung.

$$D^L(t,x) = Te^{\int_{t_0}^t dt\ \alpha' \otimes \mathcal{P}^L}(x), \quad \mathcal{P}^L(z) = \delta_{z=1} \ln\delta + \theta_{1-z>\delta} \frac{1+z^2}{2(1-z)},$$

$$D_0^L(t,x) = e^{-S}\delta(1-x),$$

$$D_1^L(t,x) = e^{-S} \int_{t_0}^t dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi}\ \alpha' \mathcal{P}_\theta^L(x),$$

$$D_2^L(t,x) = e^{-S} \int_{t_0}^t dt_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{t_0}^{t_1} dt_1 \int_0^{2\pi} \frac{d\phi_2}{2\pi}\ \alpha'^2\ \mathcal{P}_\theta^L \otimes \mathcal{P}_\theta^L(x),$$

$$D_3^L(t,x) = \dots$$

$$S = (t-t_0)\alpha' \left(\ln\frac{1}{\delta} + \frac{3}{4}\right), \quad (f\otimes g)(x) \equiv \int dydz\delta_{x=yz} f(y)g(z), \quad \alpha' = \frac{C_F \alpha_S}{\pi}.$$

Markovian with integrated LO+NLO kernel

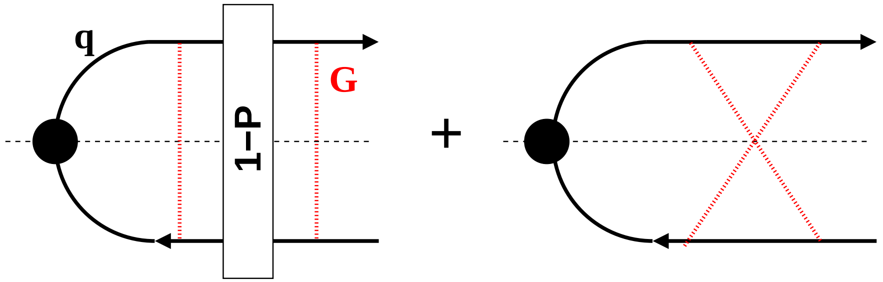
iEVMC = Markovian with N+L integrated kernel:

- As before, $k^T =$ evolution variable + mapping into standard LIPS,
- But L+N kernel $\mathcal{P}^{L+N}(\alpha', z) = \mathcal{P}^{L}(z) + \alpha'\mathcal{P}^{N}(z)$, gluonstrahlung.
- NLO part $\mathcal{P}^{N}(z)$ is the integrated/inclusive DGLAP kernel, C_F-part.

$$D_1^{L+O}(t,x) = e^{-S} \int_{t_0}^{t} dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \left(\alpha'\mathcal{P}_\theta^L(x) + {\alpha'}^2\mathcal{P}^N(x)\right),$$

$$D_2^{L+O}(t,x) = e^{-S} \int_{t_0}^{t} dt_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{t_0}^{t_1} dt_1 \int_0^{2\pi} \frac{d\phi_2}{2\pi}\, {\alpha'}^2\, \mathcal{P}_\theta^{L+N} \otimes \mathcal{P}_\theta^{L+N}(x),$$

$\mathcal{P}^{N}(\alpha', z)$ we have re-calculated from two Feynman diagrams (only real)



following methodology of Curci+Furmanski+Petronzio (1980):

$$\mathcal{P}^N = \frac{1+3x^2}{16(1-x)} \ln^2(x) + \frac{2-x}{4}\ln(x) + \frac{3}{8}(1-x).$$

NEW!!! MC with Unintegrated LO+NLO kernel

eEVMC = Markovian with N+L Unintegrated kernel:

- As before k^T as evolution variable and mapping into standard LIPS,
- But NLO part of evol. DGLAP kernel kept in UNINTEGRATED form, all over the 2-gluon LIPS!

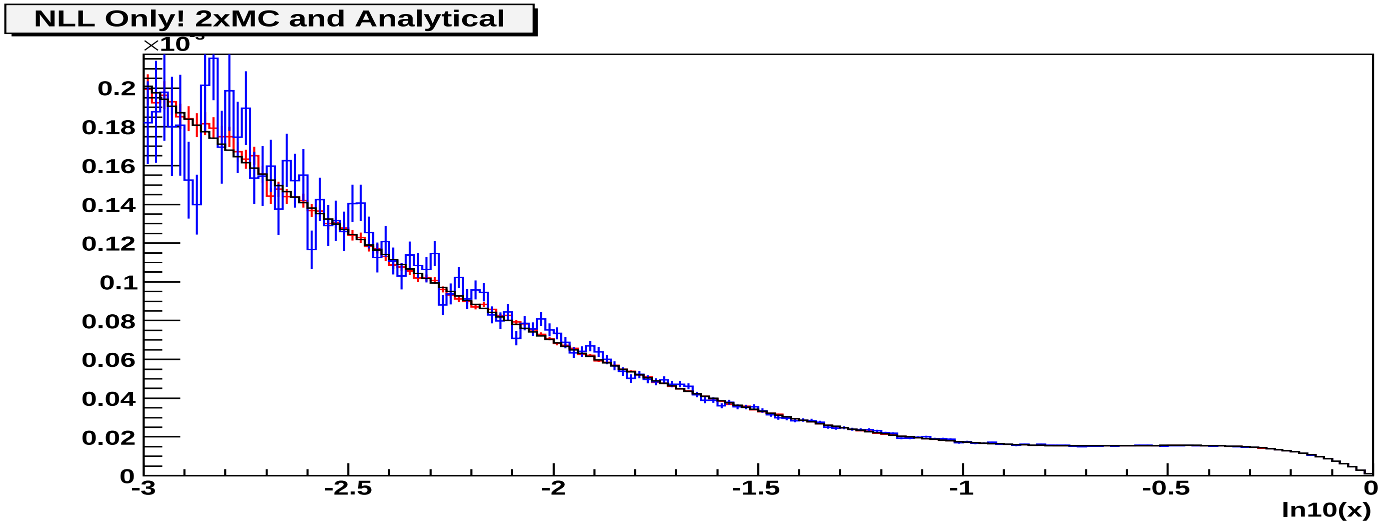
$$D_1^{L+N}(t,x) = e^{-S} \int_{t_0}^{t} dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \, \alpha' \mathcal{P}_\theta^L(x),$$

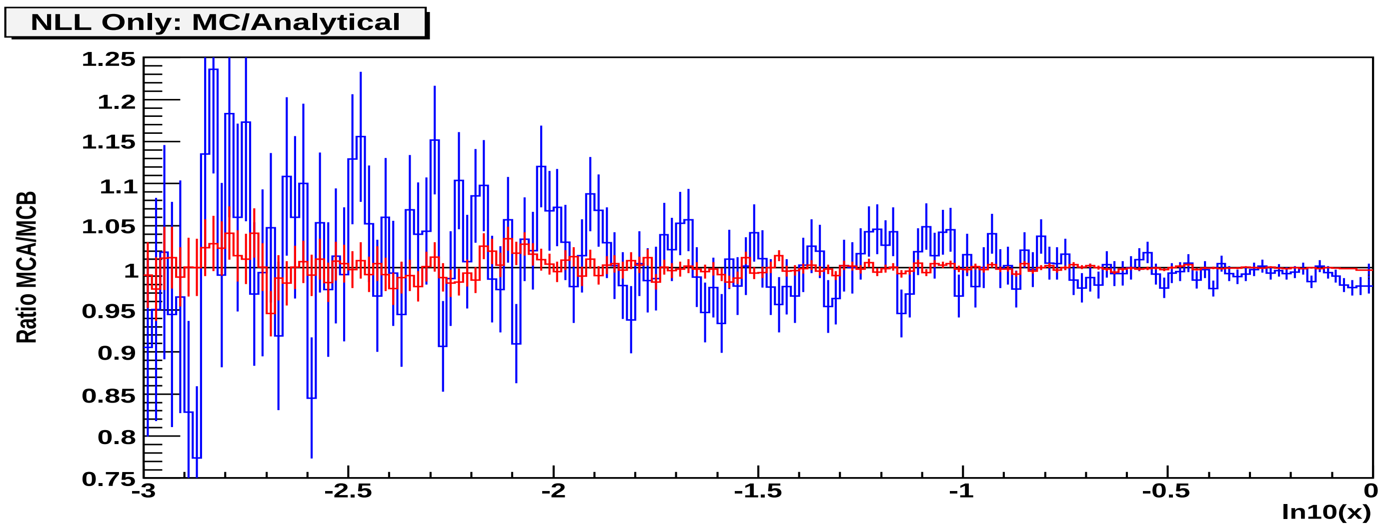
$$D_2^{L+N}(t,x) = e^{-S} \int_{e^{t_0}}^{e^t} \frac{dk_2^T}{k_2^T} \int_0^{2\pi} \frac{d\phi_2}{2\pi} \int_{e^{t_0}}^{k_2^T} \frac{dk_1^T}{k_1^T} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$

$$\int_{x=z_1 z_2} dz_1 dz_2 \; \left[\alpha'^2 \mathcal{P}^L(z_1) \mathcal{P}^L(z_2) + b_2^N(k_1^\mu, k_2^\mu) \right]$$

where NLO part $b_2^N(k_1^\mu, k_2^\mu)$ comes directly from Feynman diagrams $\times$ LIPS.
Hence **eEVMC** is (Matrix Element) **ME-based**.

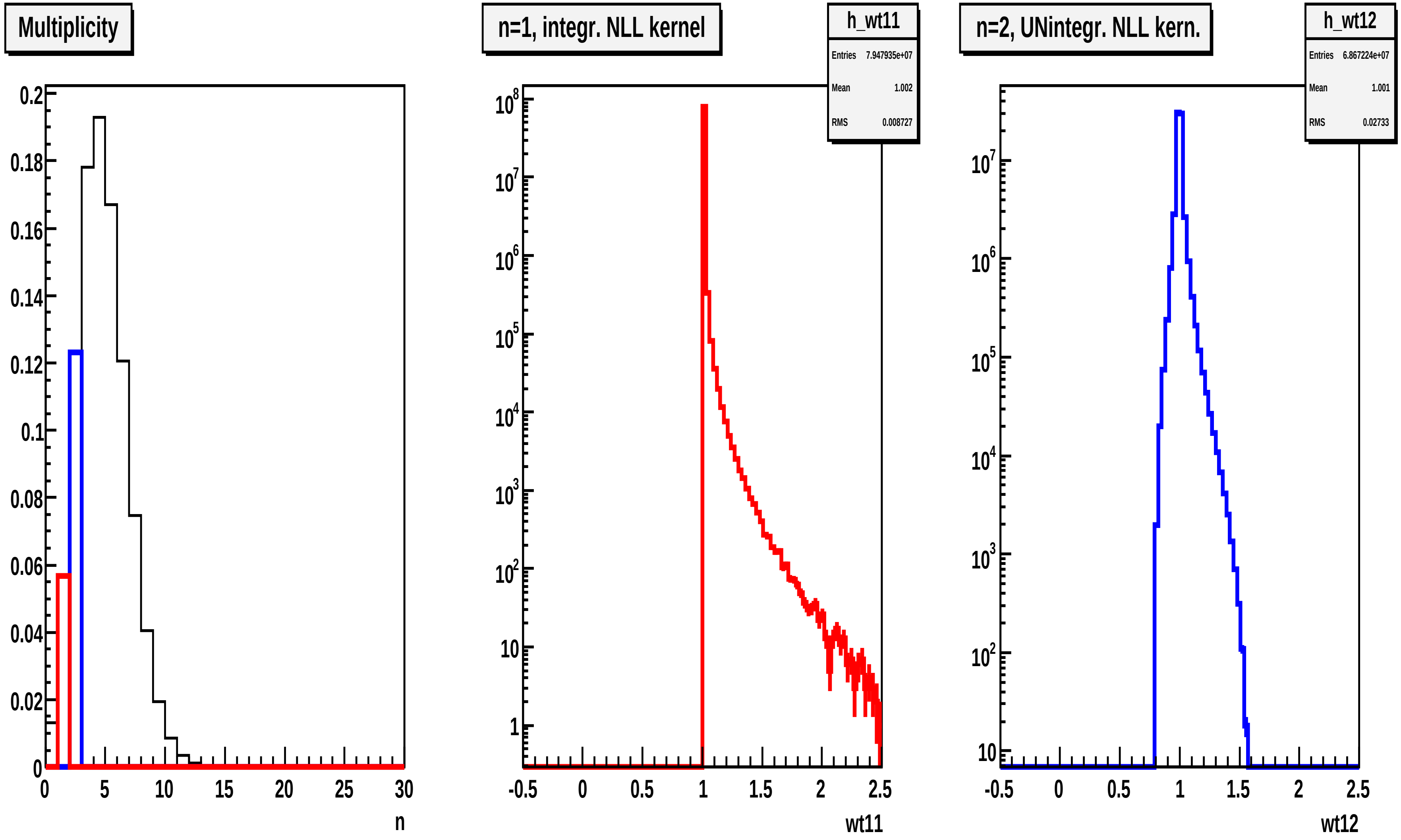
Benchmark comparison of 2 MCs and Analytical integration





- **BLACK** Analytical formula from Curci-Furmanski-Petronzio (table 1)
- **RED** Integrated kernel in Markovian MC (iEVMC), n=1!
- **BLUE** UNINTEGRATED kernel in Markovian MC (eEVMC), n=2!

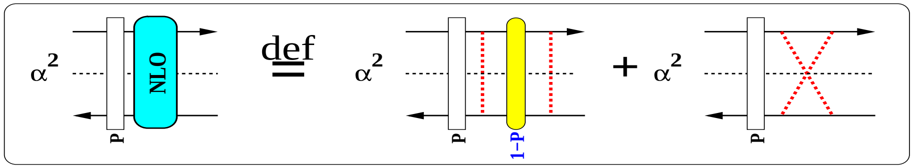
Weight distributions for NLO in the MC

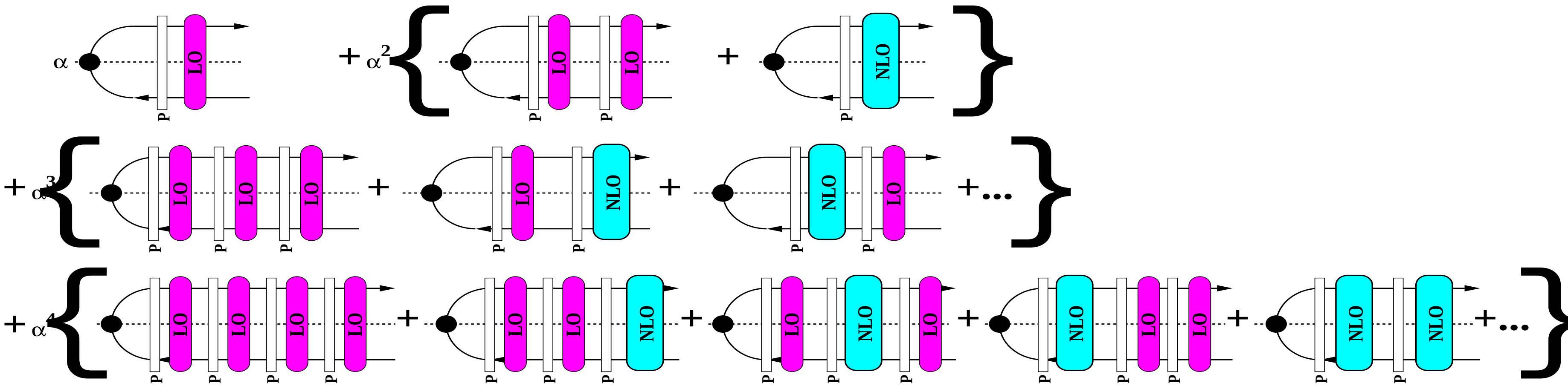


Unintegrated kernel (blue) yields more regular weight distribution!

Weight is positive and has small variance.

What next? NLO insertion for $n = 3, 4, ..\infty$





NLO decomposition in powers of α in factorization theorems. Example $n = 4$:

$$D_4^{L+N}(t,x) = e^{-S} \left(\prod_{i=1}^{4} \int \frac{d^3 k_i}{2k_i^0} \theta_{t_{i+1}>t_i} \right) \delta_{1-x=\sum_{i=1}^{3} \alpha_i} \; \rho_4^{L+N}(k_4, k_3, k_2, k_1),$$

$$\rho_4^{L+N}(k_4, k_3, k_2, k_1) = \rho^L(k_4|x_3)\; \rho^L(k_3|x_2)\; \rho^L(k_2|x_1)\; \rho^L(k_1|x_0)$$
$$+ \rho^L(k_4|x_3)\; \rho^L(k_3|x_2)\; b_2^N(k_2, k_1|x_0) + \rho^L(k_4|x_3)\; b_2^N(k_3, k_2|x_1)\; \rho^L(k_1|x_0)$$
$$+ b_2^N(k_4, k_3|x_2)\; \rho^L(k_2|x_1)\; \rho^L(k_1|x_0)$$
$$+ b_2^N(k_4, k_3|x_2)\; b_2^N(k_2, k_1|x_0)$$

Discussion, Conclusions

- Unintegrated NLO kernel within full 2-particle LIPS in the MC can be constructed.
- Dimensional regularization can be removed.
- The integrand of the NLO kernel features nice IR cancellations, such that only **short range correlation** remain for large y_i and α_i. No long tails! No cancellations between distant regions in the LIPS!
- Re-insertion of the NLO unintegrated kernel into LO MC model representing LO+NLO (DGLAP) evolution done for $n = 2$ and is perfectly feasible for $n > 2$.
- Monte Carlo weight looks regular/positive.
- A decisive/critical milestone towards NLO parton shower MC has been reached.