

Constrained Monte Carlo algorithm for the HERWIG evolution

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Evolution in Cracow, QCD

Monte Carlo modeling of $\bar{M}S$ DGLAP evolution:

- Markovian (forward) precision ($\sim 10^{-3}$) solutions of the full LL DGLAP equations (massless quarks). Acta.Phys.Pol. B35 (2004).
- Markovian precision solutions of the full NLL DGLAP equations (massless quarks). IFJPAN-V-04-08.
- Markovian study of the CCFM one-loop evolution IFJPAN-V-05-03.

Constrained Monte Carlo algorithms for DGLAP evolution:

- Constrained MC (non-Markovian) class II. Proc. Loops&Legs 2004, Nucl. Phys. Proc. Suppl. 135 (2004) and IFJPAN-V-04-06.
- Constrained MC (non-Markovian) class I. October 2004 talk at HERA-LHC wshop and IFJPAN-V-04-07.

People involved:

- K.Golec-Biernat, S.Jadach, W.Płaczek, M.Skrzypek, Z.Źas

Towards the parton shower (this talk):

- Constrained MC algorithm (class I) for HERWIG-style evolution.

Motivation and background

Known facts:

- Markovian MC implementing the QCD/QED evolution equations is the underlying ingredient in all parton shower type MCs
- Unconstrained forward Markovian MC, with evolution kernels from perturbative QCD/QED, inefficient for ISR.
- Backward evolution MC algorithm of Sjöstrand (Phys.Lett. 157B, 1985) is a widely adopted workaround.
- Backward Markovian MC does not solve the QCD evolution eqs. It merely exploits their solutions coming from the external non-MC methods

The old-standing problem:

- Is it possible to invent an efficient MC algorithm, solving internally the evolution eqs. by its own? No use of external PDFs.
- THE ANSWER IS YES! As shown in works listed on the previous page.

Motivation:

- Better modeling the ISR parton shower, possibly more friendly for inclusion of NLL and NNLL into parton shower MCs.
- Possibly easier MC modeling of the unintegrated parton distributions $D_k(p_T, x)$ and CCFM class of the QCD calculations/models.

Vocabulary

Markovian MC algorithm

The algorithm in which the number of emission (determining the dimension of the dimension of the integral, phase space), is generated as the last variable

non-Markovian MC algorithm

The algorithm in which the number of emission (the dimension of the integral), is generated as one of the first variables.

Constrained MC algorithm = CMC

The distributions are the same as in normal Markovian evolution, but the final energy $x = \prod z_i$ and the parton type $k = G, q_j, \bar{q}_j$ are predefined i.e. constrained.

HERWIG Evolution (terminology by P. Nason) :

Two ingredients:

$\alpha_S(Q(1-z))$ (Amati+Basetto+Ciafaloni+Marchesini+Veneziano, NPB173, 1980)

and $\varepsilon_{IR} = Q_0/Q$ where $Q_0 \sim 1\text{GeV}$ (Webber+Marchesini, NPB310, 1988).

For simplicity Q_0 coincides with the starting point of the QCD evolution.

MS-bar DGLAP evolution \neq HERWIG evolution

At the LL they differ by large NLL and Q_0/Q terms.

The difference going away at the NLL (Amati et.al.)

Discussion

- We have got efficient CMC algorithm (October talk) for the MS-bar DGLAP evolution.
- Is it much more difficult to extend it to HERWIG Evolution?
- In principle not. However, the CMC algorithm is quite complicated and we don't really know, until it is actually done.
- The key points to check are MC efficiency and numerical stability.
- Pure bremsstrahlung is the critical part of the CMC algorithm.
- We are going to check its efficiency for the HERWIG Evolution. We shall show that it works well, for pure bremsstrahlung.
- The rest is modeling of (up to four) Quark \leftrightarrow Gluon transitions.
- In the CMC class I Quark \leftrightarrow Gluon transitions are modeled using general purpose MC tool FOAM, hence it should work almost automatically. Still to be checked.

Pure bremsstrahlung from the “emitter” $k = G, q, \bar{q}$ line

Iterative solution of the QCD evolution equations,
for evolution $t_0 \rightarrow t$, where $t = \ln Q$ is the evolution time:

$$x\mathcal{D}_{kk}(t, t_0; x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^n \int_{t_0}^t dt_i \int_0^1 dz_i \mathcal{P}_{kk}^{\ominus}(t_i, z_i) \delta_{x=\prod_{i=1}^n z_i} \right\},$$

Notation:

- $\theta_{x>y} = 1$ for $x > y$ and $= 0$ otherwise.
- $\delta_{x=y} \equiv \delta(x - y)$.
- $\mathcal{P}_{kk}(t, z) \equiv \frac{\alpha(t, z)}{\pi} z P_{kk}(t) = -\mathcal{P}_{kk}^{\delta}(t) \delta_{z=1} + \mathcal{P}_{kk}^{\ominus}(t, z)$.
- $\mathcal{P}_{kk}^{\ominus}(t, z) = \mathcal{P}_{kk}(t, z) \theta_{1-z>\varepsilon(t)}$, the same as in LL DGLAP.
- $\mathcal{P}_{kk}^{\delta}(t) = \int_0^{1-\varepsilon(t)} dz \mathcal{P}_{kk}^{\ominus}(z, t)$, from energy sum rule, valid up to NLL.
- Sudakov formfactor: $\Phi_k(t, t_0) = \int_{t_0}^t dt' \mathcal{P}_{kk}^{\delta}(t')$.
- IR cut $\varepsilon(t) = Q_0/Q$; it is not anymore $\ll 1$, as in the standard DGLAP.

Variable mapping more complicated than for normal DGLAP

$$\int_x^{1-\varepsilon(t)} dz_i \int_{t_0}^t dt_i \mathcal{P}_{kk}^\ominus(t_i, z_i) = h_k \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \int_0^1 ds_i, \quad i = 1, 2, \dots, n,$$

$$z_i(y_i) = 1 - \exp(\rho^{-1}(y_i)),$$

$$\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i).$$

where

$$\rho(v) \equiv (\hat{t} + v) \ln(\hat{t} + v) - v - v \ln \hat{t}_0 - \hat{t} \ln \hat{t}, \quad \hat{t} \equiv t - t_\Lambda = \ln Q - \ln \Lambda_0.$$

IMPORTANT: ρ^{-1} is not analytical!

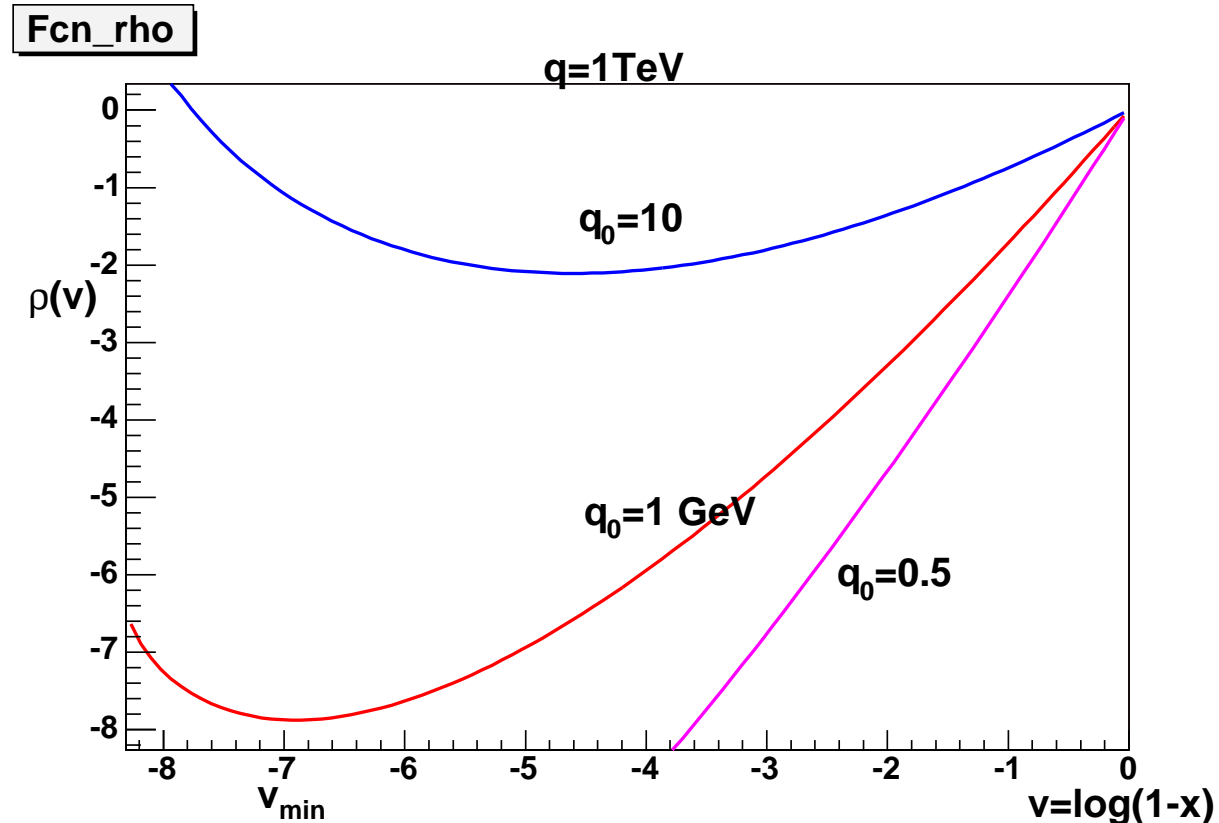
Inversion has to be done numerically. ρ^{-1} will enter the constraint function $\prod z_i$!

The above mapping leads to:

$$x \mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} h_k^n \prod_{i=1}^n \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \delta_{x=\prod_{i=1}^n z_i(y_i)} \int_0^1 ds_i \right\}.$$

Variable mapping

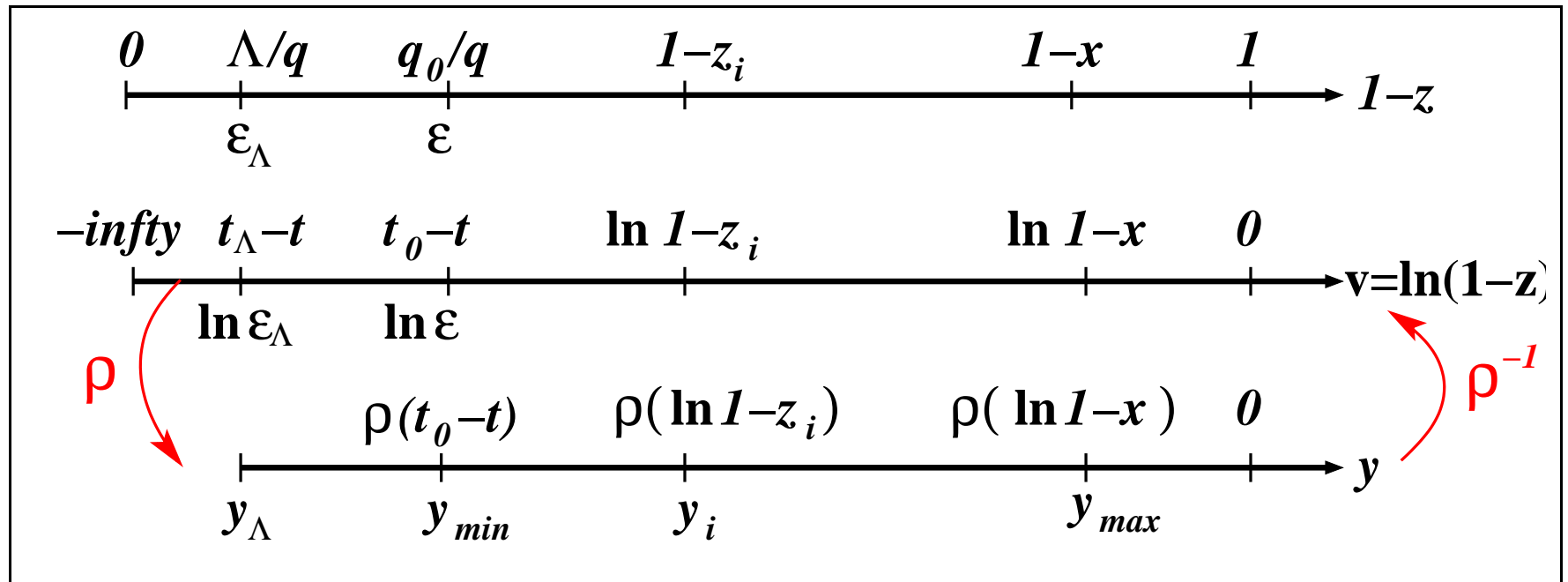
The middle (red) curve in following figure illustrates the shape of function $\rho(v)$ for $Q = 1\text{TeV}$ and $Q_0 = 1\text{GeV}$:



The minimum position is at $v_{\min} = \ln(Q_0/Q)$ where $v = \ln(1 - z)$. We only define the mapping $\rho(v)$ only for $v > v_{\min}$.

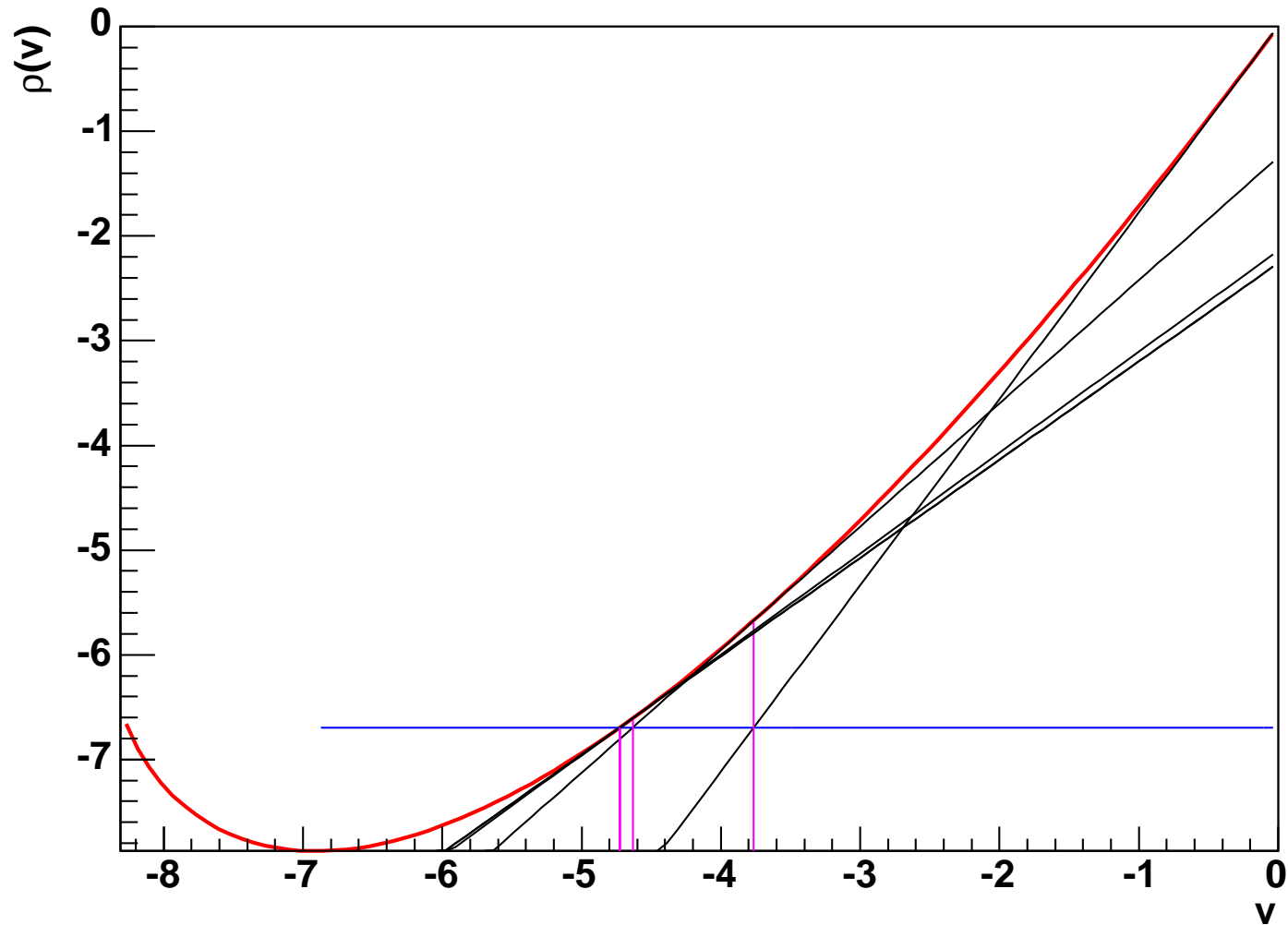
Variable mapping

Closer look into mapping of z into $v = v(z) = \ln(1 - z)$ and further into another variable $\rho(v(z))$:



Solving ρ^{-1} numerically

Red curve is the function $\rho(v)$ for $Q_0 = 1\text{GeV}$, $Q = 100\text{GeV}$, together with the illustration of the iterative method of finding $v = \rho^{-1}(y)$ using method of tangential (starting at $v = 0$).

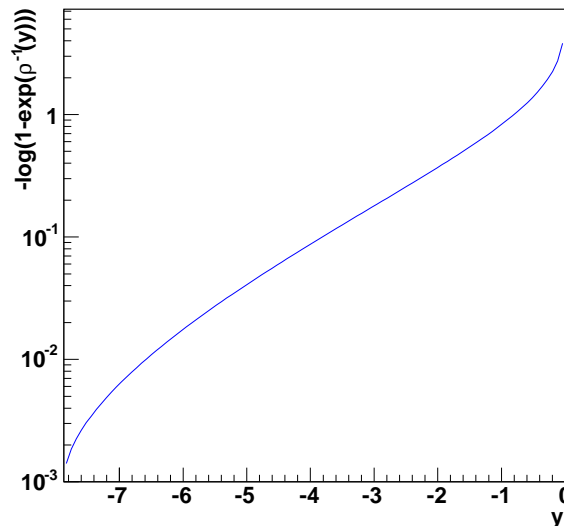


The energy constraint

Using symmetry of the integrand we finally trade the ordering in evolution time variables t_i into ordering in the energy variables y_i ($y_0 \equiv 0$):

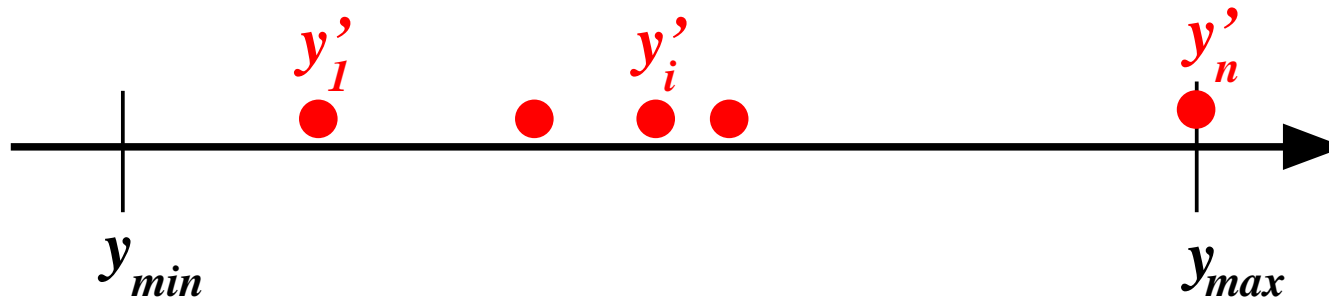
$$x\mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \right. \\ \left. + x^{-1} \sum_{n=1}^{\infty} h_k^n \prod_{i=1}^n \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \theta_{y_i > y_{i-1}} \delta \left(\ln \frac{1}{x} - \sum_j f(y_j) \right) \int_0^1 ds_i \right\}.$$

Here, $f(y_i)$ is very steeply (exponentially) rising, see plot below for $q_0 = 1\text{GeV}$ and $q = 1000\text{GeV}$



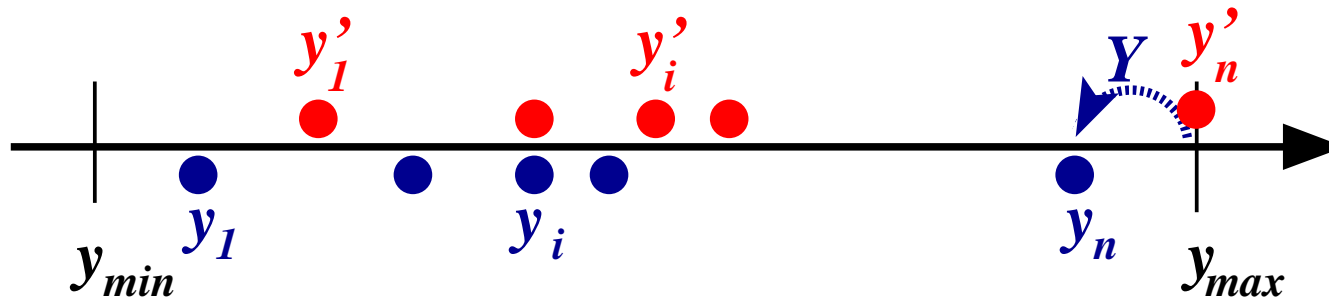
hence the constraint $x = \prod_{i=1}^n z_i(y_i)$ is “saturated” by a single z .

Linear shift: $y'_i \rightarrow y_i = y'_i - Y$



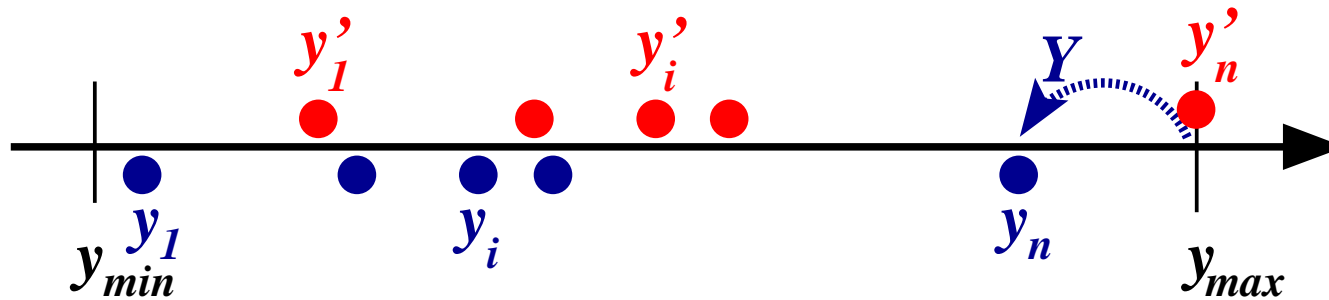
- Begin with y'_i such that one of them $y_n \equiv y_{max}$

Linear shift: $y'_i \rightarrow y_i = y'_i - Y$



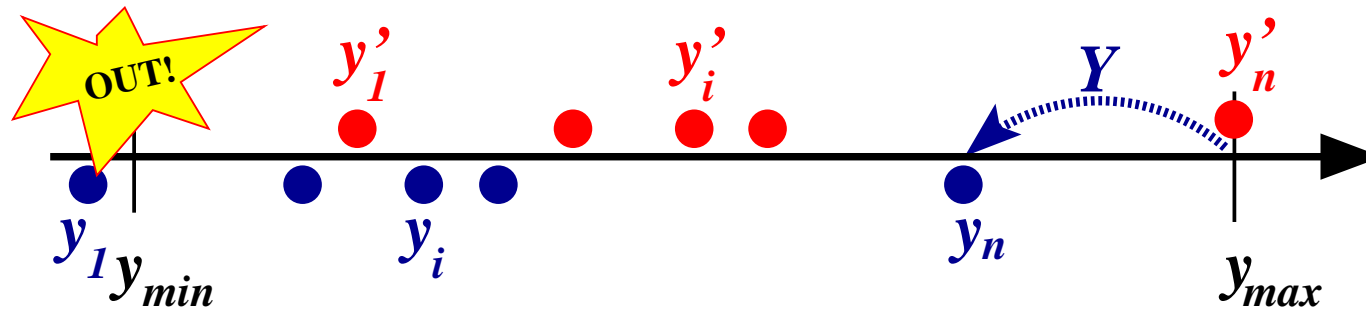
- Begin with y'_i such that one of them $y_n \equiv y_{max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$

Linear shift: $y'_i \rightarrow y_i = y'_i - Y(y'_1, y'_2, \dots, y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i

Linear shift: $y'_i \rightarrow y_i = y'_i - Y(y'_1, y'_2, \dots, y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit y_{\min} . Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung Monte Carlo

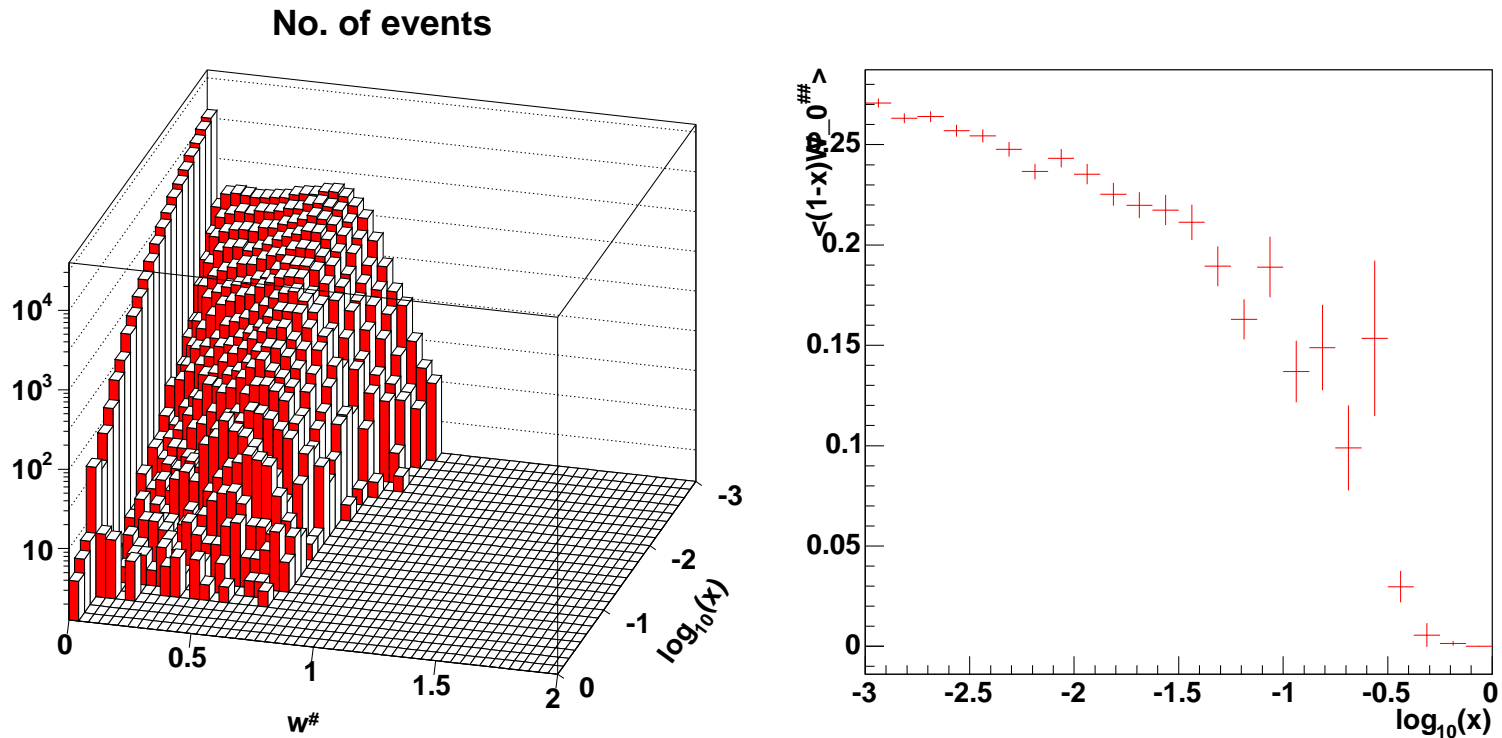
$$x \mathcal{D}_{kk}(\tau, \tau_0; x) = e^{(\tau - \tau_0) a_k} \sum_{n=0}^{\infty} \left\{ e^{b_k \mathcal{R}(\varepsilon)} \delta_{n=0} \delta_{x=1} + \delta_{n>0} \theta_{1-x>\varepsilon} e^{b_k \mathcal{R}(1-x)} \frac{b_k x^{\omega_k - 1}}{x g(x)} \right. \\ \left. \times P_n(b_k [\mathcal{R}(1-x) - \mathcal{R}(\varepsilon)]) \prod_{i=1}^n \int_0^1 dr_i \frac{\delta(1 - \max r_j)}{n} \int_0^1 ds_i w^\# \right\}$$

NOTATION:

- Mapping $z_i(y_i) = 1 - \exp(\rho^{-1}(y_i))$.
- Mapping $\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i)$.
- Poisson distribution: $P_n(\lambda) = e^{-\lambda} \lambda^n / n!$, $\lambda = \langle n \rangle$.
- $\mathcal{R}(1 - z) \equiv \rho(\ln(1 - z))$, (implicitly depends on t and t_0).
- MC weight: $w^\# = w_P \frac{x g(x)}{|\partial_Y \ln F(Y_0)|} \theta_{y'_1 - Y_0 > y_{\min}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y'_i is here relaxed (to get explicit $1/(n-1)!$ of Poisson).

Prototype Monte Carlo I.d

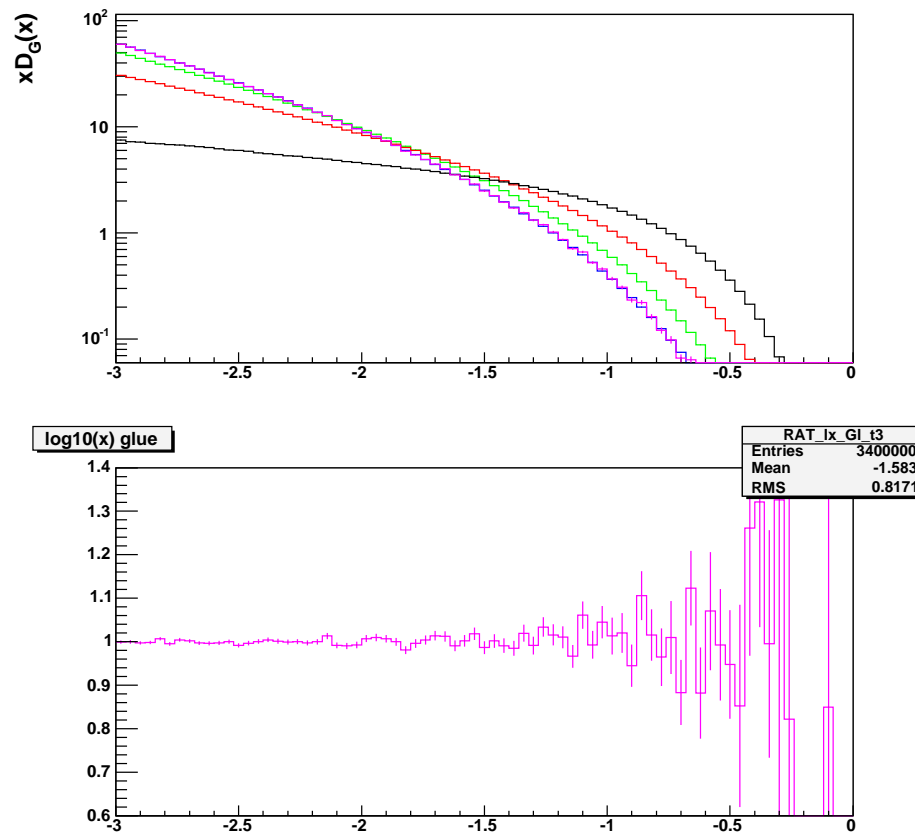
The starting parton distribution is that of gluon in the proton.



Plotted are weight distribution and the average weight as a function of x .
The maximum weight is below one and the acceptance rate 0.25 is surprisingly good!
About 1/3 of events has zero weight.

The comparison with the simpler Markovian MC

Below is the comparison of CMC I.d with the unconstrained Markovian MC for the HERWIG evolution, gluonstrahlung:



There is a reasonable agreement, within the statistical error, for small statistics, so far.

Summary

- It is demonstrated using prototype program (bremsstrahlung) that the Constrained MC works in practice for the HERWIG evolution.
- Still to be done: implementing Quark—Gluon transitions for the HERWIG evolution, as it was already done for the $\overline{\text{MS}}$ -bar DGLAP
- Including the rest of NLL corrections into CMC,
- and more...
- However, most difficult technical problems in constructing Constrained MCs for DGLAP-like evolutions are solved!