

QCD evolution using Monte Carlo methods

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Introduction and outline

- Experiments at Large Hadron Collider will offer good quality very high statistics data (millions of events) on the production of the W and Z bosons.
- It will be highly nontrivial task to exploit them fully in order to measure very precisely mass of W, anomalous couplings, find signals of the presence of new physics at multi-TeV scales, measure precisely parton luminosities etc.
- For this purpose one needs a new class of the Monte Carlo tools which incorporate high quality, beyond-the-leading-order QCD, QED and Electroweak calculations, which do not exist yet.
- I shall describe new method of MC modeling of QCD evolution, which will hopefully lead us to a new brave world of the combined QCD \otimes EW Monte Carlo calculations for LHC.
- Keywords: LHC, Standard Model, QCD, precision Monte Carlo

Expectations for EW observables at LHC

NOW → **LHC**:

$$\delta M_W: \quad 34\text{MeV} \rightarrow 15\text{MeV} \quad (d\sigma/dM_T^l)$$

$$\Delta\kappa_\gamma, \Delta\lambda_\gamma: \quad 1 \cdot 10^{-2} \rightarrow 3 \cdot 10^{-3} \quad (pp \rightarrow VV)$$

$$\delta \sin_{\text{eff}}^2: \quad 17 \cdot 10^{-5} \rightarrow 15 \cdot 10^{-5} \quad (pp \rightarrow Z)$$

$$\delta M_t: \quad 5\text{GeV} \rightarrow 1\text{GeV} \quad (ljjb)$$

- M_W increasingly important player in the SM precision tests
- Anomalous vector boson couplings constrained factor 10 better
- EW mixing angle measured as precisely as in LEP (??!!)
- Parton luminosities to be measured with 1% precision

Desired specs of TOOLS for EW observables at LHC

Observable \in process	EW	QED	QCD	MC type
$M_W \in W, Z$	Impr. Born	$\mathcal{O}(\alpha)_{\text{exp}}$ FSR!	pdf(x,pT), NLO?	events!
Anom. coupl. $\in VV$	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	NLO! NNLO?	events!
$\sin^2 \theta_{EW} \in Z$	$\mathcal{O}(\alpha, \alpha_{Sud.}^2)$	$\mathcal{O}(\alpha)_{\text{exp}}$ FSR!	NLO	events!
Parton $\mathcal{L} \in W, Z$	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)_{\text{exp}}$ FSR?	NLO! NNLO?	events?

! \equiv mandatory,

? \equiv to be checked...

None of the existing TOOLS fulfill the above specifications

Existing EW+QCD tools for $pp \rightarrow V$ and $pp \rightarrow VV$, where $V=W,Z$

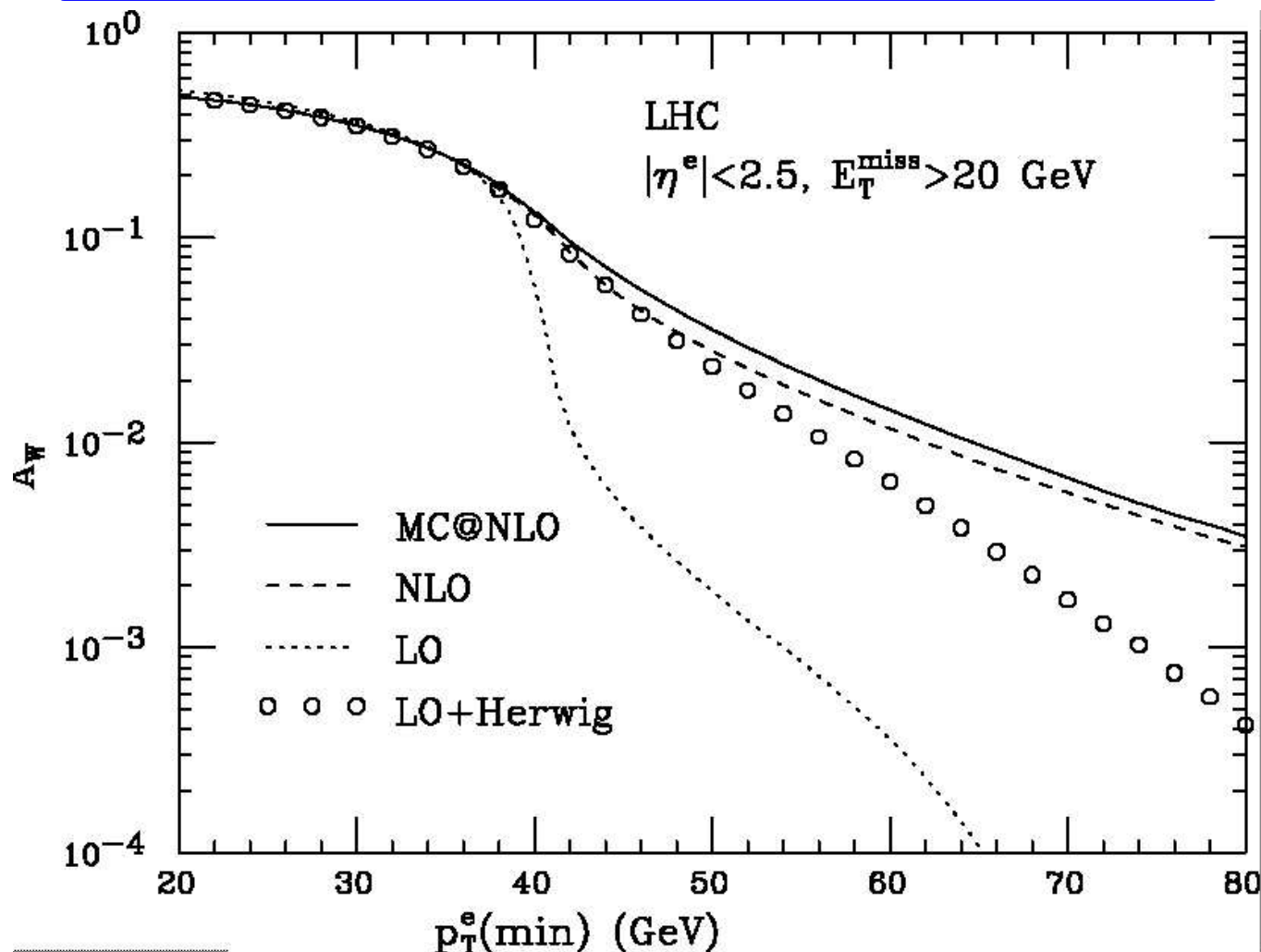
Required at least $\mathcal{O}(\alpha)$ electroweak or NLO QCD, none has both!

Tool	Process	EW	QCD	MC type
WGRAD	W	$\mathcal{O}(\alpha)$	pdf(x),LO	histogrs.
ZGRAD2	Z	$\mathcal{O}(\alpha)$	pdf(x),LO	histogrs.
WINHAC	W	QED FSR $\mathcal{O}(\alpha)_{\text{EEX}}$	pdf(x),LO	events
HORACE	W,Z	QED FSR part.sh.	pdf(x),LO	events
SANC	W,Z	$\mathcal{O}(\alpha)$???	events?
RESBOS	W, Z	LO	pdf(x,pT),NLO	histogrs.
DYRAD	V+(0j-1j)	LO	pdf(x),NLO	histogrs.
MCFM	V,VV	LO	pdf(x),NLO	histogrs.
DKS	WW, WZ, ZZ	LO, Anom.Coup.	pdf(x),NLO	histogrs.
dFS	$W\gamma, Z\gamma$	LO, Anom.Coup.	pdf(x),NLO	histogrs.
MC@NLO	WW	LO	part.sh. NLO	events
MC@NLO	W or Z	LO	part.sh. NLO	events

Conclusions of the EW group MC4LHC workshop, August 2003

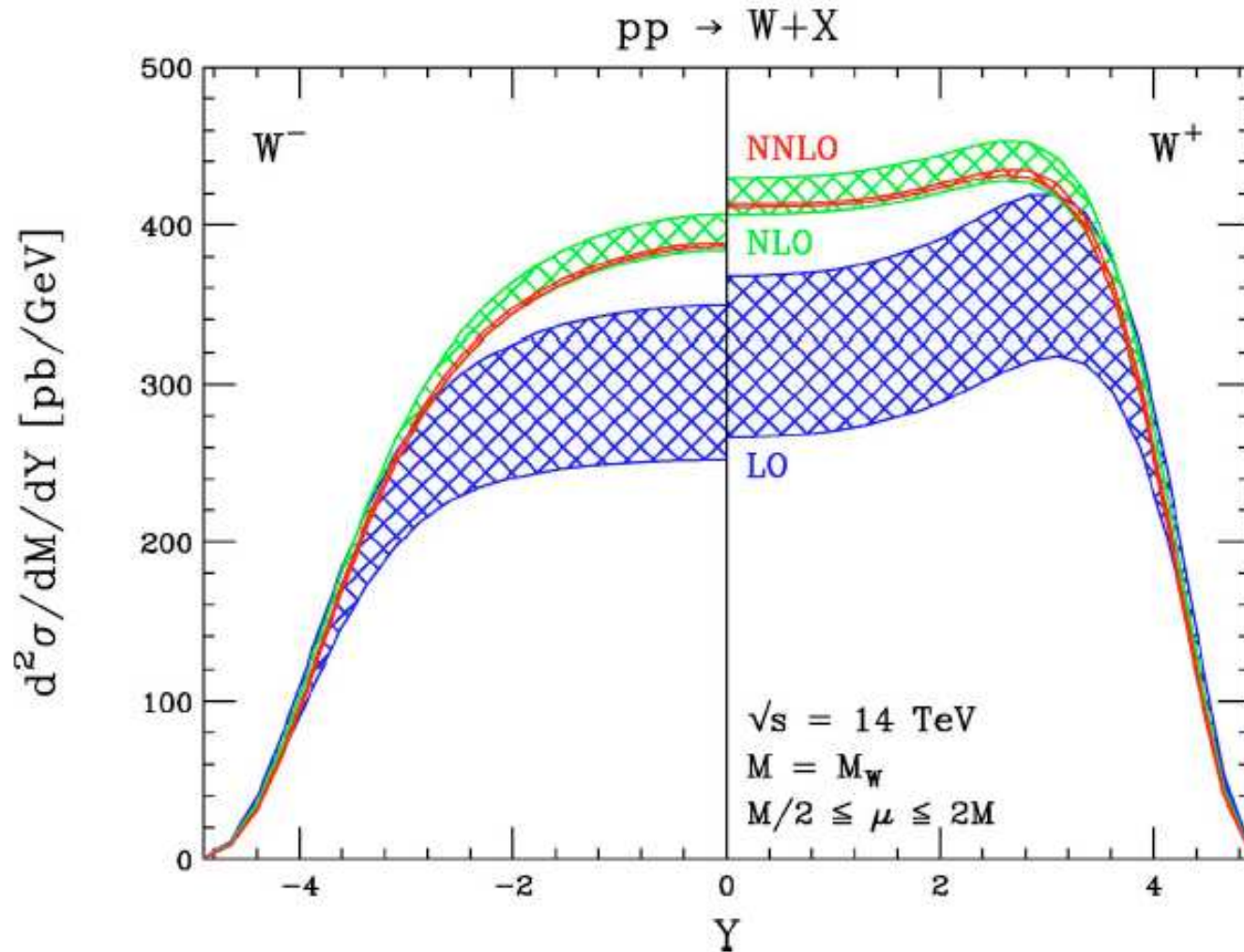
- In spite of the rich legacy of “Tevatron codes”, it is fair to say that there is no single (MC) TOOL fully adequate for any EW observable nor parton luminosity measurement at LHC.
- New MCs combining first order EW and QCD seem to appear on the horizon.
- Plans for systematic test of tools for EW@LHC should materialize.

Recent QCD NLO calculations (2003/4)



- Parton shower NLO: Frixione and Mangano using MC@NLO (JHEP05, 2004, 056)
 - For $p_e^T > 20\text{GeV}$ and $|\eta_e| < 2.5$ QCD theory error $\delta\sigma/\sigma \geq 2\%$.
 - Parton shower $\sim 1\%$, NNLO $\sim 1\%$, PDF $\sim 1\%$ (determination of lumi!)
 - Spin effects crucial, QED FSR to be added.
 - What about δM_W ?

Recent QCD NNLO calculations (2003/4)

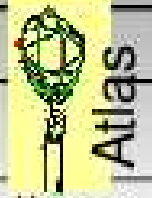


- C. Anastasiou et.al. Phys.Rev.D69:094008,2004, W rapidity distr. at NNLO
- Estimated QCD theoretical errors $\leq 1\%$, fantastic!!!
 - Calculation too inclusive for the practical applications.
 - No QED ISR, even LL.

M_W uncertainty, ATLAS study of the error budget (2000)

Expected error budget of the W -mass measurement at LHC

<u>Source</u>	<u>CDF Run Ib</u>	<u>ATLAS or CMS</u>	$W \rightarrow l \nu$, one lepton species
	30K evts, 84 pb ⁻¹	60M evts, 10fb ⁻¹	
Statistics	65 MeV	< 2 MeV	
Lepton scale	75 MeV	15 MeV	most serious challenge
Energy resolution	25 MeV	5 MeV	known to 1.5% from Z peak
Recoil model	33 MeV	5 MeV	scales with Z statistics
W width	10 MeV	7 MeV	$\Delta\Gamma_W \approx 30$ MeV (Run II)
PDF	15 MeV	10 MeV	
Radiative decays	20 MeV	< 10 MeV	(improved Theory calc)
$P_T(W)$	45 MeV	5 MeV	$P_T(Z)$ from data, $P_T(W)/P_T(Z)$ from theory
Background	5 MeV	5 MeV	
<u>TOTAL</u>	113 MeV	≤ 25MeV	Per expt, per lepton species



M_W uncertainty, new ATLAS study (2005)

Measurement of the W boson mass?

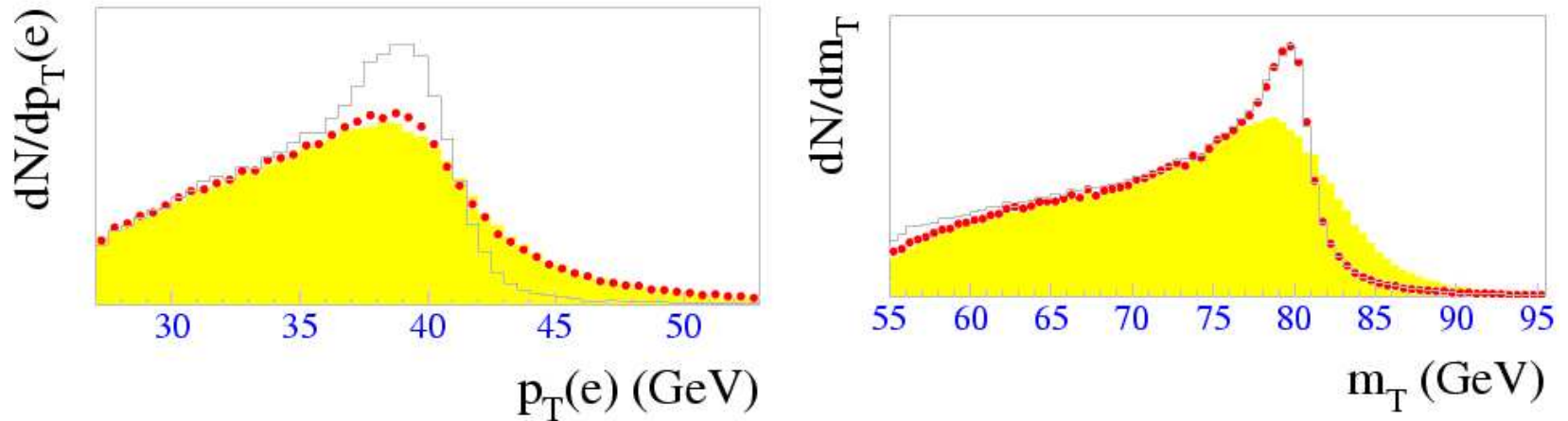
ATLAS Rome Workshop, 6th of June 2005

Where we are heading		
Source	Knowledge	Uncert. [MeV]
PDF _(Theo/Exp)	current error (CTEQ6)	~ 20
E-p scale _(Exp)	$\sim 1 \times 10^{-4}$	~ 10
Resolution _(Exp)	~ 1%	~ 5
Background _(Exp)	if ignored	~ 5*
Recoil Model (p_T^W) _(Theo)	use Z recoil & p_T^W / p_T^Z	~ 6
Radiative decays _(Theo/Exp)	PHOTOS	< 10*
W width _(Exp)	straight forward variation	~ 7*
Lepton ID/Jet rejection _(Exp)		small?
Material _(Exp)	Mapped with $\gamma \rightarrow ee, Z$	
Pile-up & Underlying Event	Measure in Z^0 events	

*) From TDR

- Error budget in the future ATLAS measurement of M_W , June2005.

How is M_W measured at Tevatron?



- Transv. mass $M_T = (2p_T^l p_T^\nu (1 - \cos\phi_{l\nu}))^{1/2}$ unaffected by smearing due to p_T^W . (Plot is from hep-ph/0108272; Red dots: added p_T^W , Yellow: added detector eff.)
- Mass of W is fitted in Tevatron from the M_T distribution.
- In M_T distr. uncertainty due to p_T^W enters back through experim. cuts, hence the net gain is in fact small. **Perfect detector needed (simulation).**
- Use of p_T^l distr. is reconsidered. **Perfect theory needed.**
- Distribution of p_T^W is modeled using data on leptonic decays of Z. LHC will profit here from Z statistics much larger than in Tevatron.
- New methods: scaling M_T distribution from Z decay, MC templates etc.
- The biggest EW/QED effect is from QED FSR, see next slide.

Where are we?

- Complete MC tools for EW+QCD precision ($\sim 1\%$) predictions for LHC still not available.
Partial incomplete solutions come close to specs.
- Progress done in a recent couple of years, for “semi-inclusive” QCD observables, NNLO PDFs, NNLO distributions, matching NLO with parton shower, etc.
- More effort needed to implement QCD+QED predictions in form of the **high quality exclusive MC tools**
(MC simulators providing weight one events)
- More effort to create precision “numerical benchmarks”, elaborate more detailed evaluation/estimation of theoretical errors, etc.

Perturbative calculations in QCD

Matrix element, QFT textbook approach:

- Finite (fixed) order calculation $\left[\sum_N \int d\text{LIPS}_N |M_N|^2 \right]$, where
- N = Number of real emissions determined in the beginning LIPS = Lorentz Invariant Phase Space
- The above is convoluted with the strictly collinear PDFs.

Classic Parton shower approach:

- Emission chain built by resurrecting transverse momenta in the LL QCD evolution equations
- MC algorithm of the Markovian type, N determined in the end
- Hard scattering at the Born level
- Very much improved LL approximation but always incomplete NLL in the shower tree and in the hard process ME

Hybrid approaches:

- Andre+Sjöstrand, Frixione+Webber, Nason and more...
- CKKW (Catani et.al.)
- The unsolved problem: NLL parton shower. Hopeless?

Vocabulary

Markovian MC algorithm

The algorithm in which the number of emission (determining the dimension of the dimension of the integral, phase space), is generated as the last variable

non-Markovian MC algorithm

The algorithm in which the number of emission (the dimension of the integral), is generated as one of the first variables.

Constrained MC algorithm = CMC

The distributions are the same as in normal Markovian evolution, but the final energy $x = \prod z_i$ and the parton type $k = G, q_j, \bar{q}_j$ are predefined i.e. constrained.

HERWIG Evolution (terminology by P. Nason), 1-loop CCFM :

Two ingredients:

$\alpha_S(Q(1-z))$ (Amati+Basetto+Ciafaloni+Marchesini+Veneziano, NPB173, 1980)

and $\varepsilon_{IR} = Q_0/Q$ where $Q_0 \sim 1\text{GeV}$ (Webber+Marchesini, NPB310, 1988).

For simplicity Q_0 coincides with the starting point of the QCD evolution.

MS-bar DGLAP evolution \neq HERWIG evolution

At the LL they differ by large NLL and Q_0/Q terms.

The difference going away at the NLL (Amati et.al., Trentadue+Kodaira, Catani et.al.)

Monte Carlo modeling of the QCD \overline{MS} DGLAP evolution:

- Markovian MC (forward) precision ($\sim 10^{-3}$) solutions of the full LL DGLAP equations (massless quarks). Acta.Phys.Pol. B35 (2004).
- Markovian MC precision solutions of the full NLL DGLAP equations (massless quarks). IFJPAN-V-04-08, to appear.
- Markovian MC study of the CCFM one-loop evolution. IFJPAN-V-05-03, to appear

Constrained Monte Carlo (CMC) algorithms for DGLAP evolution:

- Constrained MC (non-Markovian) class II. Proc. Loops&Legs 2004, Nucl. Phys. Proc. Suppl. 135 (2004) and IFJPAN-V-04-06, hep-ph/0504205.
- Constrained MC (non-Markovian) class I. October 2004 talk at HERA-LHC wshop and IFJPAN-V-04-07, hep-ph/0504263.

People involved:

- K.Golec-Biernat, S.Jadach, W.Płaczek, M.Skrzypek, Z.Was

Towards the parton shower (this talk):

- **Constrained MC algorithm (class I) for the HERWIG-style evolution, also referred to as CCFM one-loop. See also talk at HERA-LHC wshop, March 05.**
- NB. Presented CMC techniques are quite general. The problem of constraining the longitudinal momentum in ISR parton shower (QCD and QED) is always present, whichever scenario we may imagine for the future development!

Motivation and background

Known facts:

- Markovian MC implementing the QCD/QED evolution equations is the underlying ingredient in all parton shower type MCs
- Unconstrained forward Markovian MC, with evolution kernels from perturbative QCD/QED, inefficient for ISR.
- Backward evolution MC algorithm of Sjöstrand (Phys.Lett. 157B, 1985) is a widely adopted workaround.
- Backward Markovian MC does not solve the QCD evolution eqs. It borrows solutions from the external non-MC calculations, stored in look-up tables of PDF(Q, x).

The old-standing problem:

- Is it possible to invent an efficient MC algorithm, solving internally the evolution eqs. by its own? No use of external PDFs.
- THE ANSWER IS YES! As shown in works listed on the previous page.

Motivation:

- Better modeling the ISR parton shower, possibly more friendly for inclusion of NLL and NNLL into parton shower MCs.
- Possibly easier MC modeling of the unintegrated parton distributions $D_k(p_T, x)$ and CCFM class of the QCD calculations/models.

First results

- The first efficient CMC algorithm (October 2004) was found for the MS-bar DGLAP evolution.
- Pure bremsstrahlung is the critical part of the CMC algorithm.
- Presently we are upgrading bremsstrahlung part of CMC from DGLAP to HERWIG evolution and this bremsstrahlung version will be presented here.
- We find that the efficiency of CMC for the HERWIG Evolution is satisfactory.
- The rest is modeling of (up to four) Quark \leftrightarrow Gluon transitions; this is originally done for DGLAP, now extended to HERWIG Evolution.
- In the CMC class I Quark \leftrightarrow Gluon transitions are modeled using general purpose MC tool FOAM.
- In the following we show essential part of the CMC algorithm for pure bremsstrahlung and show briefly results for full the DGLAP/HERWIG evolution with Quark \leftrightarrow Gluon transitions.

Pure bremsstrahlung from the “emitter” $k = G, q, \bar{q}$ line

Iterative solution of the QCD evolution equations,
for evolution $t_0 \rightarrow t$, where $t = \ln Q$ is the evolution time:

$$x\mathcal{D}_{kk}(t, t_0; x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^n \int_{t_0}^t dt_i \int_0^1 dz_i \mathcal{P}_{kk}^{\ominus}(t_i, z_i) \delta_{x=\prod_{i=1}^n z_i} \right\},$$

Notation:

- $\theta_{x>y} = 1$ for $x > y$ and $= 0$ otherwise.
- $\delta_{x=y} \equiv \delta(x - y)$.
- $\mathcal{P}_{kk}(t, z) \equiv \frac{\alpha(t, z)}{\pi} z P_{kk}(t) = -\mathcal{P}_{kk}^{\delta}(t) \delta_{z=1} + \mathcal{P}_{kk}^{\ominus}(t, z)$.
- $\mathcal{P}_{kk}^{\ominus}(t, z) = \mathcal{P}_{kk}(t, z) \theta_{1-z>\varepsilon(t)}$, the same as in LL DGLAP.
- $\mathcal{P}_{kk}^{\delta}(t) = \int_0^{1-\varepsilon(t)} dz \mathcal{P}_{kk}^{\ominus}(z, t)$, from energy sum rule, valid up to NLL.
- Sudakov formfactor: $\Phi_k(t, t_0) = \int_{t_0}^t dt' \mathcal{P}_{kk}^{\delta}(t')$.
- IR cut $\varepsilon(t) = Q_0/Q$; it is not anymore $\ll 1$, as in the standard DGLAP.

Variable mapping more complicated than for normal DGLAP

$$\int_x^{1-\varepsilon(t)} dz_i \int_{t_0}^t dt_i \mathcal{P}_{kk}^\Theta(t_i, z_i) = h_k \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \int_0^1 ds_i \mathbf{1}, \quad i = 1, 2, \dots, n,$$

$$z_i(y_i) = 1 - \exp(\rho^{-1}(y_i)),$$

$$\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i).$$

where

$$\rho(v) \equiv (\hat{t} + v) \ln(\hat{t} + v) - v - v \ln \hat{t}_0 - \hat{t} \ln \hat{t}, \quad \hat{t} \equiv t - t_\Lambda = \ln Q - \ln \Lambda_0.$$

IMPORTANT: ρ^{-1} is not analytical!

Inversion has to be done numerically. ρ^{-1} will enter the constraint function $\prod z_i!$

The above mapping leads to:

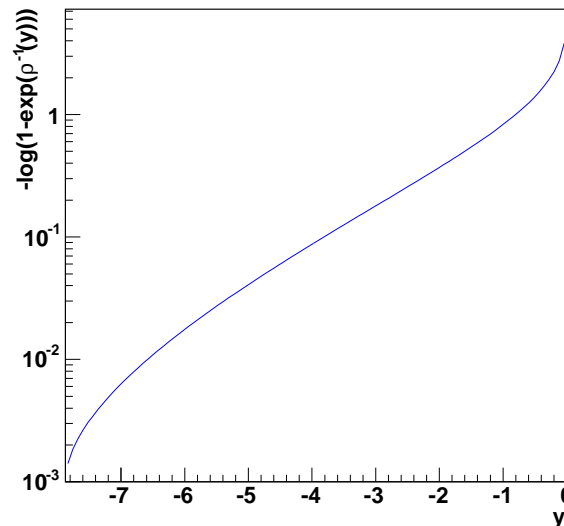
$$x \mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} h_k^n \prod_{i=1}^n \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \delta_{x=\prod_{i=1}^n z_i(y_i)} \int_0^1 ds_i \right\}.$$

The energy constraint

Using symmetry of the integrand we finally trade the ordering in evolution time variables t_i into ordering in the energy variables y_i ($y_0 \equiv 0$):

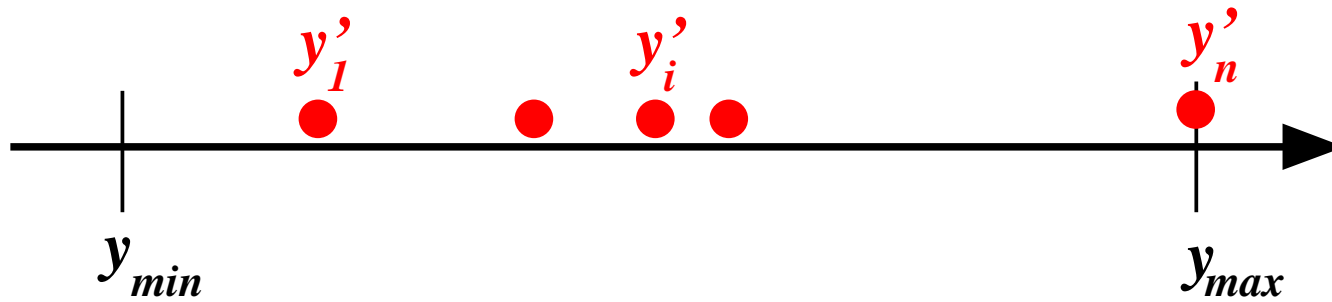
$$x \mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t, t_0)} \left\{ \delta_{x=1} + \right. \\ \left. + x^{-1} \sum_{n=1}^{\infty} h_k^n \prod_{i=1}^n \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \theta_{y_i > y_{i-1}} \delta \left(\ln \frac{1}{x} - \sum_j f(y_j) \right) \int_0^1 ds_i \right\}.$$

Here, $f(y_i)$ is very steeply (exponentially) rising, see plot below for $q_0 = 1\text{GeV}$ and $q = 1000\text{GeV}$



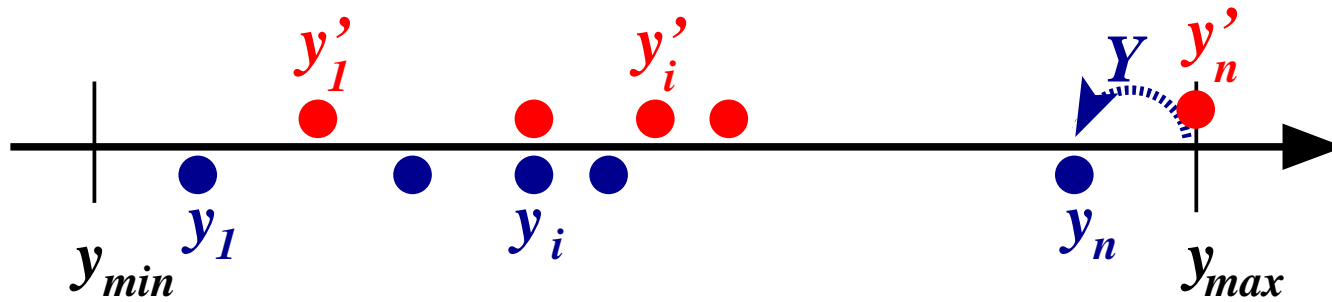
hence the constraint $x = \prod_{i=1}^n z_i(y_i)$ is “saturated” by a single z .

Linear shift: $y'_i \rightarrow y_i = y'_i - Y$



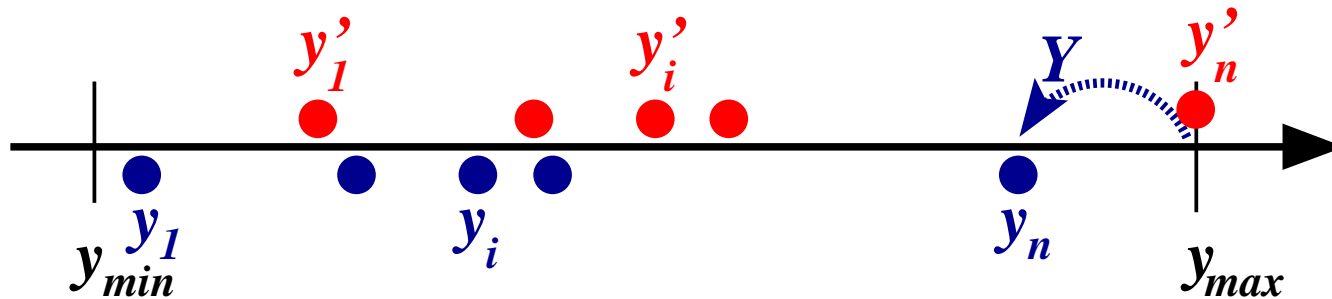
- Begin with y'_i such that one of them $y_n \equiv y_{max}$

Linear shift: $y'_i \rightarrow y_i = y'_i - Y$



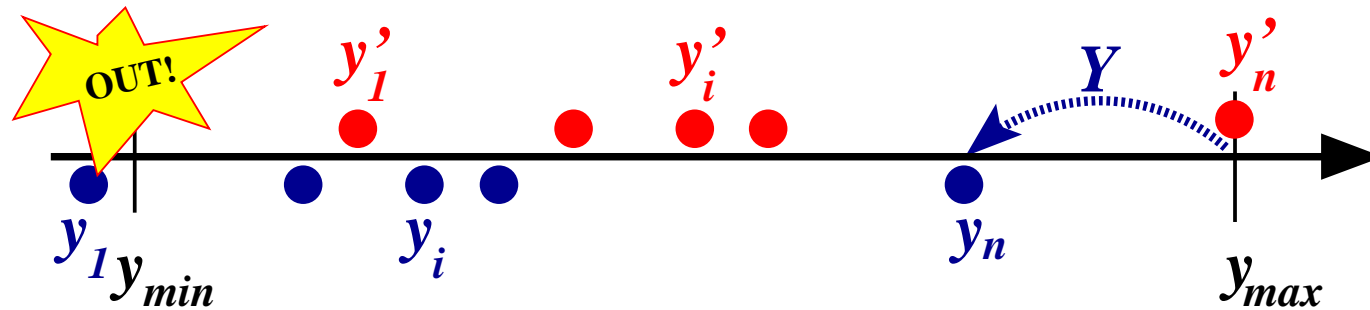
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$

$$\text{Linear shift: } y'_i \rightarrow y_i = y'_i - Y(y'_1, y'_2, \dots, y'_n)$$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i

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- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \rightarrow y_i$ by Y , where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit y_{\min} . Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung CMC

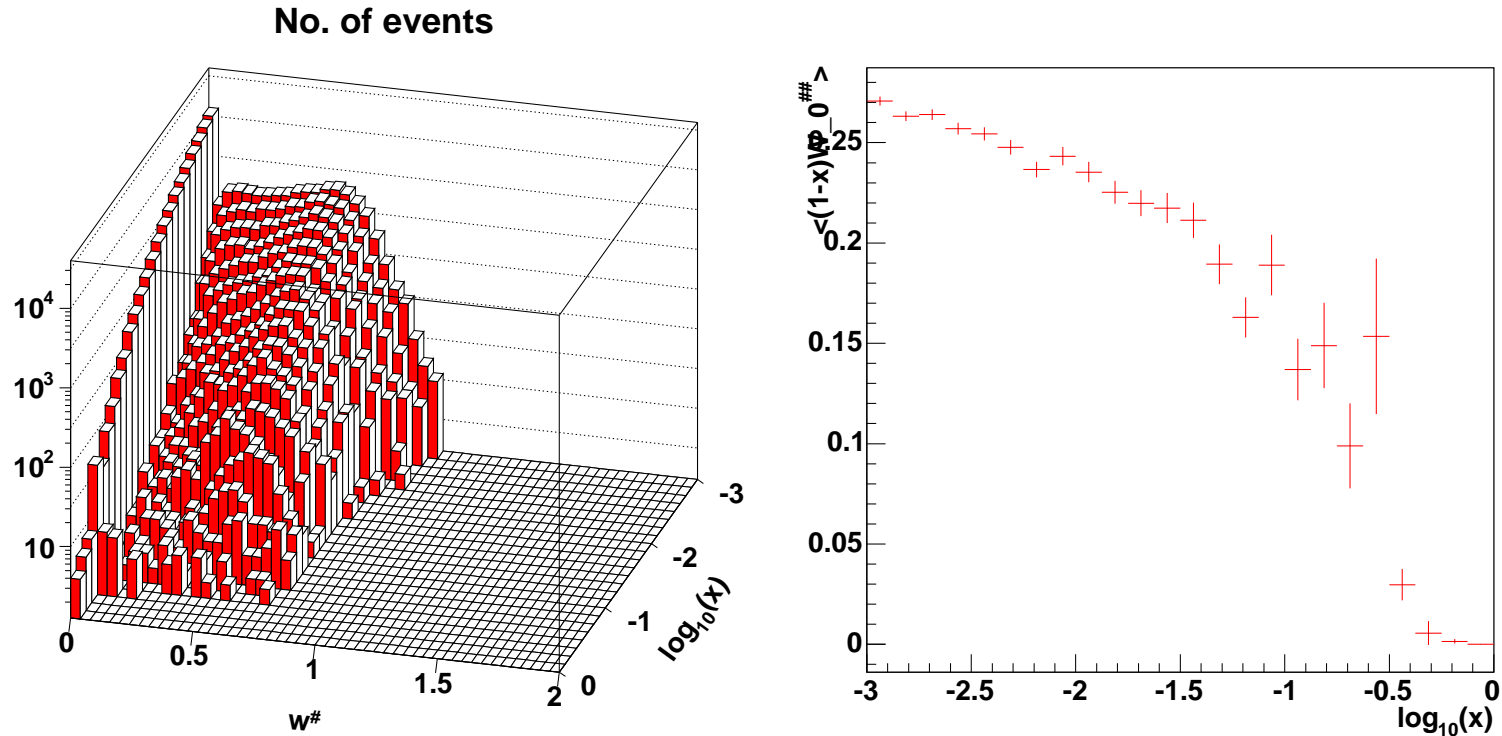
$$x \mathcal{D}_{kk}(\tau, \tau_0; x) = e^{(\tau - \tau_0) a_k} \sum_{n=0}^{\infty} \left\{ e^{b_k \mathcal{R}(\varepsilon)} \delta_{n=0} \delta_{x=1} + \delta_{n>0} \theta_{1-x>\varepsilon} e^{b_k \mathcal{R}(1-x)} \frac{b_k x^{\omega_k - 1}}{xg(x)} \right. \\ \left. \times P_n(b_k [\mathcal{R}(1-x) - \mathcal{R}(\varepsilon)]) \prod_{i=1}^n \int_0^1 dr_i \frac{\delta(1 - \max r_j)}{n} \int_0^1 ds_i w^\# \right\}$$

NOTATION:

- Mapping $z_i(y_i) = 1 - \exp(\rho^{-1}(y_i))$.
- Mapping $\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i)$.
- Poisson distribution: $P_n(\lambda) = e^{-\lambda} \lambda^n / n!$, $\lambda = \langle n \rangle$.
- $\mathcal{R}(1 - z) \equiv \rho(\ln(1 - z))$, (implicitly depends on t and t_0).
- MC weight: $w^\# = w_P \frac{xg(x)}{|\partial_Y \ln F(Y_0)|} \theta_{y'_1 - Y_0 > y_{\min}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y'_i is here relaxed (to get explicit $1/(n-1)!$ of Poisson).

Prototype Monte Carlo I.d

The starting parton distribution is that of gluon in the proton.

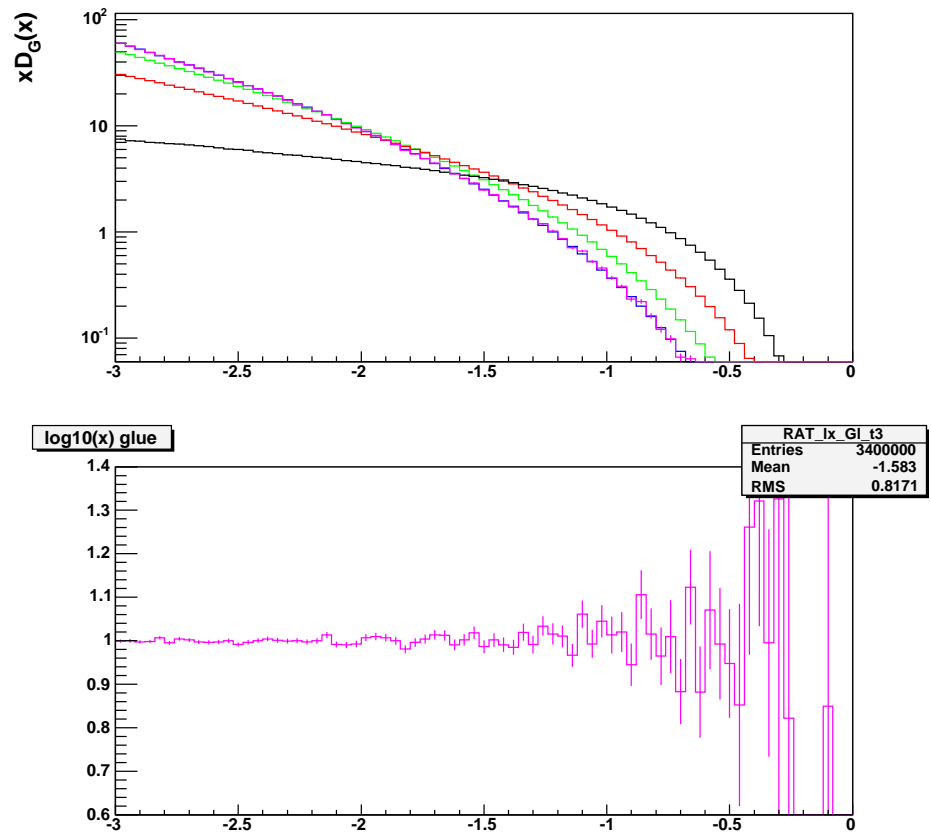


Plotted are weight distribution and the average weight as a function of x .
The maximum weight is below one and the acceptance rate 0.25 is surprisingly good!
About 1/3 of events has zero weight.

THIS IS STILL UNPUBLISHED RESULT

The comparison with the simpler Markovian MC

Below is the comparison of CMC I.d (HERWIG evolution, gluonstrahlung) with the unconstrained Markovian MC:



There is a reasonable agreement, within the statistical error, for small statistics, so far.

THIS IS STILL UNPUBLISHED RESULT

Hierarchic reorganization of the emission chain (DGLAP)

The full Constrained MC requires two-level organization of the emission chain:

(S) Flavor transmutation super-level $G \rightarrow Q \rightarrow G \rightarrow Q \rightarrow G \rightarrow \dots$

(B) Bremsstrahlung sub-level, any No. of gluon emissions ($Q \rightarrow Q, G \rightarrow G$).

Here DGLAP case is shown – HERWIG evolution is treated quite similarly.

The starting point is the usual iterative solution ($k \equiv k_n$) of the QCD evolution equations:

$$\begin{aligned}
 xD_k(\tau, x) &= e^{-(\tau-\tau_0)R_k} xD_k(\tau_0, x) + \\
 &+ \sum_{n=1}^{\infty} \sum_{k_{n-1} \dots k_1 k_0} \left[\prod_{j=1}^n \int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j > \tau_{j-1}} \right] \int_0^1 dx_0 \left[\prod_{i=1}^n \int_0^1 dz_i \right] \\
 &\times e^{-(\tau-\tau_n)R_k} \left[\prod_{i=1}^n \mathcal{P}_{k_i k_{i-1}}^{\Theta}(z_i) e^{-(\tau_i-\tau_{i-1})R_{k_{i-1}}} \right] x_0 D_{k_0}(\tau_0, x_0) \delta_{x=x_0} \prod_{i=1}^n z_i,
 \end{aligned}$$

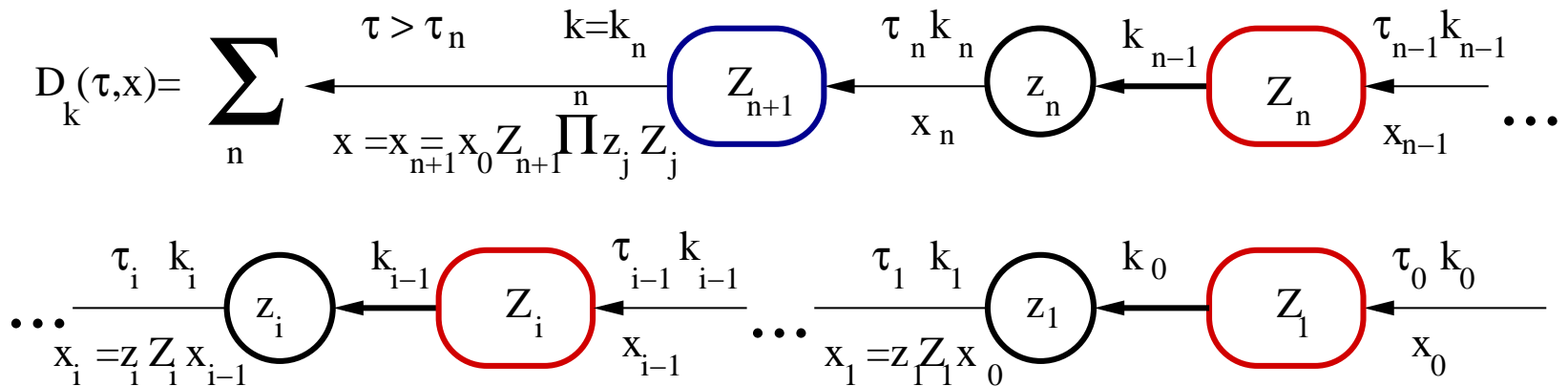
Notation:

● **Kernels:** $\mathcal{P}_{k_i k_{i-1}}^{\Theta}(z_i) = \frac{\alpha(t_0)}{\pi} z_i P_{k_i k_{i-1}}(z_i) \theta_{1-z_i > \epsilon}$

● **Transition rates:** $R_k = \sum_j \int_0^{1-\epsilon} dz \mathcal{P}_{jk}^{\Theta}(z) = \sum_j R_{jk}$

Two-level hierarchic: \mathcal{D}_{kk} are also multi-integrals!

$$\begin{aligned}
 D_k(\tau, x) &= \int dZ dx_0 \mathcal{D}_{kk}(\tau, Z|\tau_0) D_k(\tau_0, x_0) \delta_{x=Zx_0} + \\
 &+ \sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}, \dots, k_1, k_0 \\ k_n \neq k_{n-1} \neq \dots \neq k_1 \neq k_0}} \int_0^1 dZ_{n+1} \left[\prod_{j=1}^n \int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j > \tau_{j-1}} \int_0^1 dz_j \int_0^1 dZ_j \right] \int_0^1 dx_0 \\
 &\times \mathcal{D}_{kk}(\tau, Z_{n+1}|\tau_n) \left[\prod_{i=1}^n \mathbf{P}_{k_i k_{i-1}}^{\ominus}(z_i) \mathcal{D}_{k_{i-1} k_{i-1}}(\tau_i, Z_i|\tau_{i-1}) \right] \\
 &\times D_{k_0}(\tau_0, x_0) \delta\left(x - x_0 Z_{n+1} \prod_{i=1}^n z_i Z_i\right), \quad k \equiv k_n.
 \end{aligned}$$



Red oval is pure bremsstrahlung segment; **Blue oval** is $Q \leftrightarrow G$ transition.

Two-level hierarchic: \mathcal{D}_{kk} are also multi-integrals!

$$\begin{aligned}
 D_k(\tau, x) &= \int dZ dx_0 \mathcal{D}_{kk}(\tau, Z|\tau_0) D_k(\tau_0, x_0) \delta_{x=Zx_0} + \\
 &+ \sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}, \dots, k_1, k_0 \\ k_n \neq k_{n-1} \neq \dots \neq k_1 \neq k_0}} \int_0^1 dZ_{n+1} \left[\prod_{j=1}^n \int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j > \tau_{j-1}} \int_0^1 dz_j \int_0^1 dZ_j \right] \int_0^1 dx_0 \\
 &\times \mathcal{D}_{kk}(\tau, Z_{n+1}|\tau_n) \left[\prod_{i=1}^n \mathbf{P}_{k_i k_{i-1}}^{\ominus}(z_i) \mathcal{D}_{k_{i-1} k_{i-1}}(\tau_i, Z_i|\tau_{i-1}) \right] \\
 &\times D_{k_0}(\tau_0, x_0) \delta\left(x - x_0 Z_{n+1} \prod_{i=1}^n z_i Z_i\right), \quad k \equiv k_n, \\
 \mathcal{D}_{kk}(\tau, Z|\tau_0) &= \frac{e^{\Phi_k(\tau, \tau_0)}}{Z} \left\{ \delta_{Z=1} + \sum_{n=1}^{\infty} \prod_{i=1}^n \int_{\tau_0}^{\tau} d\tau_i \theta_{\tau_i > \tau_{i-1}} \int_0^1 dz_i z_i \mathbf{P}_{kk}^{\ominus}(z_i) \delta_{Z=\prod_{i=1}^n z_i} \right\}
 \end{aligned}$$

NOTATION:

- Pure brems. Sudakov formfactor $\Phi_k(\tau, \tau_0) = (\tau - \tau_0)(a_k + b_k \ln \varepsilon)$
- Kernel \times coupling const: $\mathbf{P}_{k_1 k_2}^{\ominus}(z) = \frac{2}{\beta_0} P_{k_1 k_2}(z) \theta_{1-z > \varepsilon}$
- Other: $b_k \equiv \frac{2}{\beta_0} B_{kk}, \quad a_k \equiv \frac{2}{\beta_0} A_{kk},$

CMC for full DGLAP; top level integrand for FOAM

- Neglecting temporarily $w^\#$ inside the segments \mathcal{D}_{kk} , gluon bremsstrahlung sub-level, we can integrate/sum analytically over all variables of the sub-level
- The overall (energy) x -constraint δ -function is eliminated using $\int dx_0$
- We are left with the $3n + 1$ -dim. integrals (n = No. of flavor changes) of the flavor-changing super-level, the **INTEGRAND FOR FOAM** is the following:

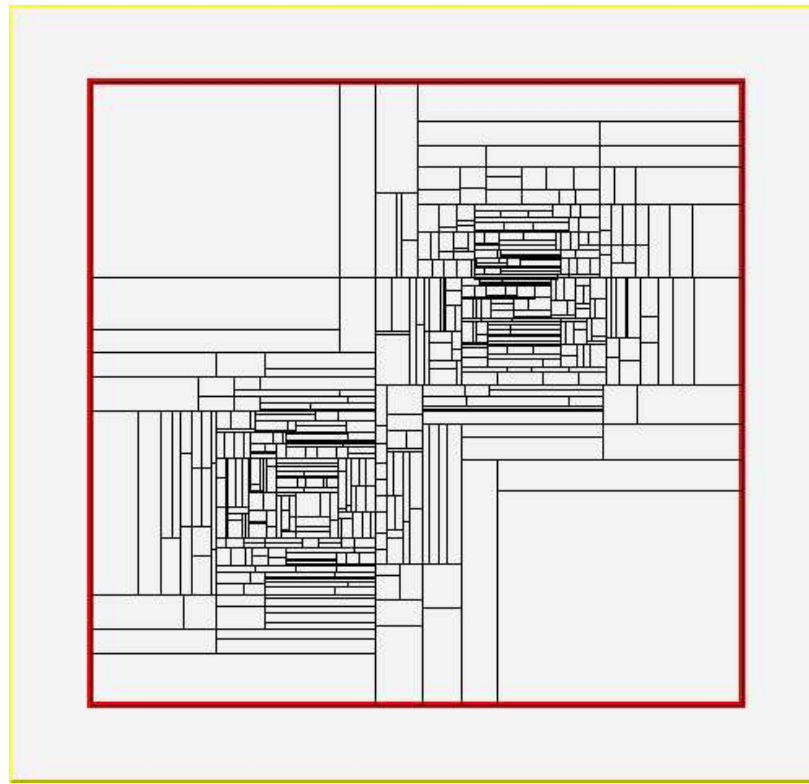
$$\begin{aligned}
 D_k(\tau, x) &= x^{-1} \int_x^1 dZ \int_0^{R(x)} dR_1 Z(R_1)^{\omega_k-2} e^{a_k(\tau-\tau_0)} x_0 D_k(\tau_0, x_0) + \\
 &+ x^{-1} \sum_{n=1}^{\infty} \sum_{\substack{k_{n-1} \dots, k_1 k_0 \\ k_n \neq k_{n-1} \neq \dots \neq k_1 \neq k_0}} \left[\prod_{j=1}^n \int_{\tau_0}^{\tau} d\tau_j \theta_{\tau_j > \tau_{j-1}} \right] \int_0^{R(x)} dR_{n+1} Z(R_{n+1})^{\omega_k-2} e^{a_k(\tau-\tau_n)} \\
 &\times \left[\prod_{i=1}^n \int_{x_{i+1}}^1 dz_i \mathbf{P}_{k_i k_{i-1}}^\ominus(z_i) \int_0^{R(x_{i+1}/z_i)} dR_i Z(R_i)^{\omega_{k_{i-1}}-2} e^{a_{k_{i-1}}(\tau_i-\tau_{i-1})} \right] \\
 &\times x_0 D_{k_0}(\tau_0, x_0), \\
 R(Z_i) &= (1 - Z_i)^{b_{k_{i-1}}(\tau_i-\tau_{i-1})}, \quad Z(R_i) = 1 - \exp\left(\left(b_{k_{i-1}}(\tau_i - \tau_{i-1})\right)^{-1} \ln R_i\right),
 \end{aligned}$$

What is FOAM for?

- Suppose you want to generate randomly points (vectors) according to an arbitrary probability distribution in n dimensions, for which you supply your own subprogram. FOAM can do it for you! Even for distributions with strong peaks and discontinuous!
- FOAM generates random points with weight one or with variable weight.
- FOAM is capable to integrate using efficient "adaptive" MC method.

How does it work?

- It creates hyper-rectangular "foam of cells", which is more dense around its peaks.
- See the following 2-dim. example of the map of 1000 cells for doubly peaked distribution:



CMC algorithm of type I, full DGLAP

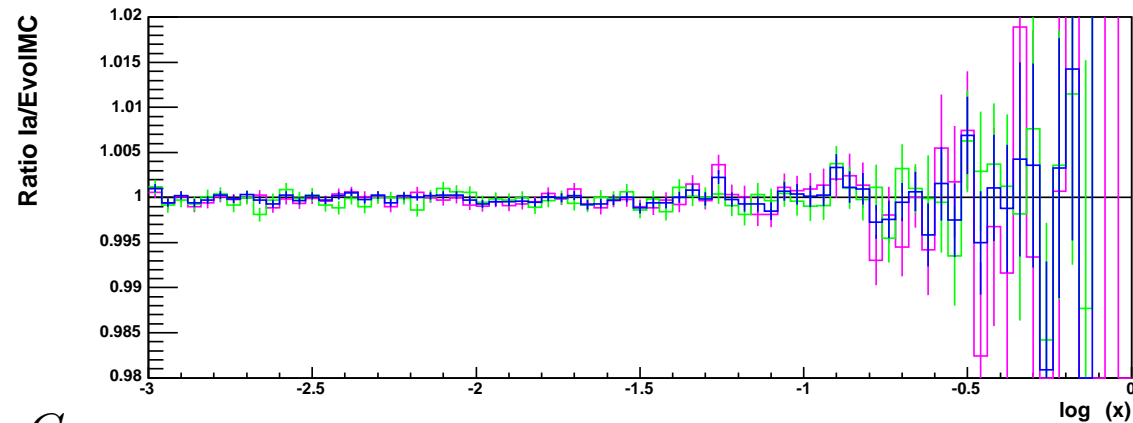
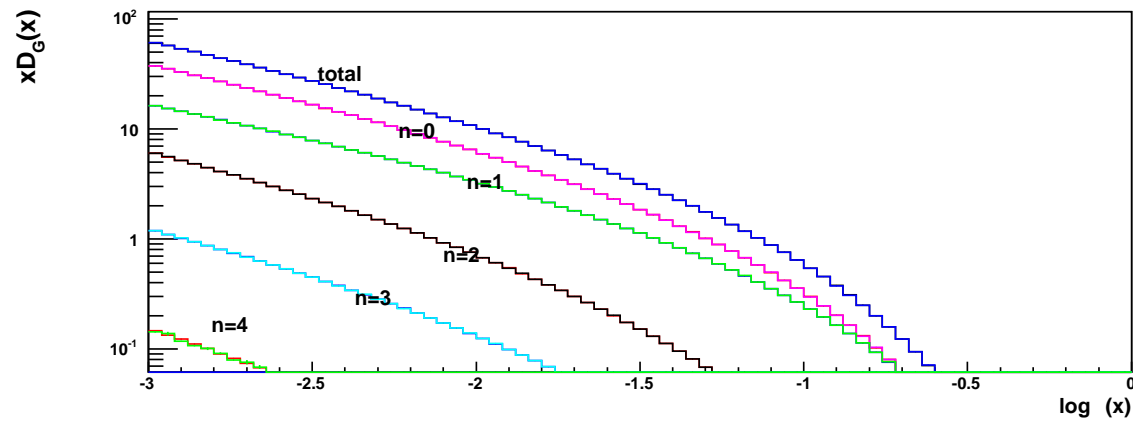
CMC algorithm description

- Generate super-level variables n, k_i, τ_i, Z_i and z_i using **Foam** general purpose MC tool.
- Limiting no. of flavor transition ($G \rightarrow Q$ and $Q \rightarrow G$) to $n = 0, 1, 2, 3$ is enough, for the $\sim 0.2\%$ precision.
- For each pure gluon bremsstrahlung segment defined by Z_i and (τ_i, τ_{i-1}) , $i = 1, 2, \dots, n + 1$, gluon emission variable $(z_j^{(i)}, \tau_j^{(i)})$, $j = 1, 2, \dots, n^{(i)}$, are generated using previously described dedicated CMC.
- Weight= 1 events available!

Numeric tests

- In the next slides we show numerical results from such a non-Markovian CMC `Evo1CMC` for “evolution” ranging from $Q = 1\text{GeV}$ to $Q = 1\text{TeV}$, $x > 10^{-3}$,
- They are compare them with the results of the Markovian unconstrained evolution of our own `Evo1FMC`
- `Evo1FMC` was previously x-checked with `QCDnum16` and `ACHEB`
- The agreement of Nonmarkovian `Evo1CMC` and Markovian `Evo1FMC` is excellent, $\sim 0.25\%$.

Test of non-Markovian Constrained MC; DGLAP



$n = 0: G \rightarrow G$

$n = 1: Q \rightarrow G$ and any no. of gluon emissions out of Q and G ,

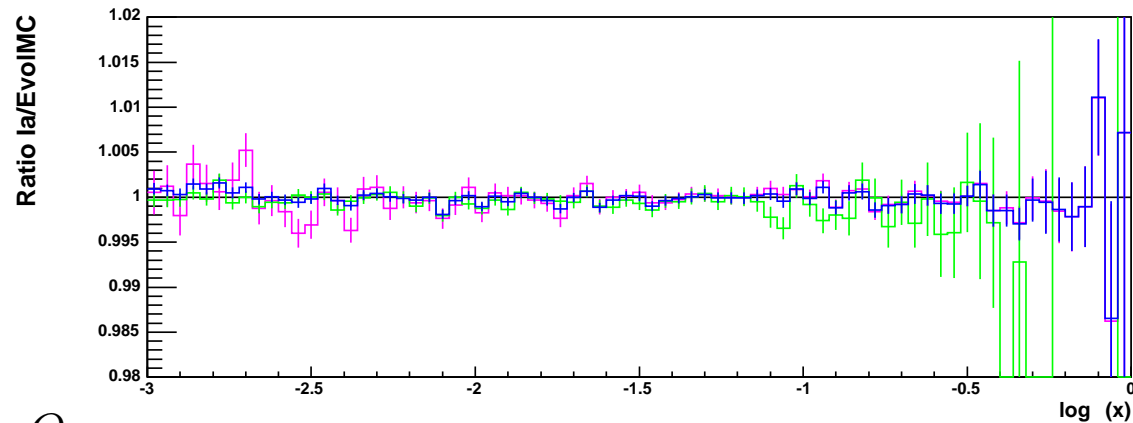
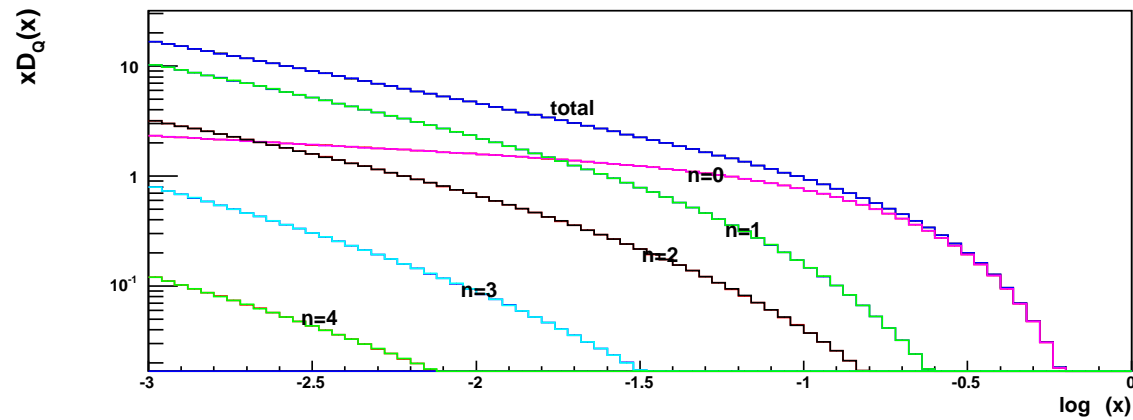
$n = 2: G \rightarrow Q \rightarrow G$, etc.

$n = 3: Q \rightarrow G \rightarrow Q \rightarrow G$, etc.

$n = 4: G \rightarrow Q \rightarrow G \rightarrow Q \rightarrow G$, etc. "Total" is the sum of $n = 0, 1, 2, 3, 4$.

Evolution from proton at $Q = 1\text{GeV}$ up to 1TeV . New non-Markovian CMC (EvoIMC) agrees with unconstrained Markovian MC (EvoIFMC) to within $\sim 0.2\%$!

Test of non-Markovian Constrained MC; DGLAP



$n = 0: Q \rightarrow Q$

$n = 1: G \rightarrow Q$ and any no. of gluon emissions out of Q and G ,

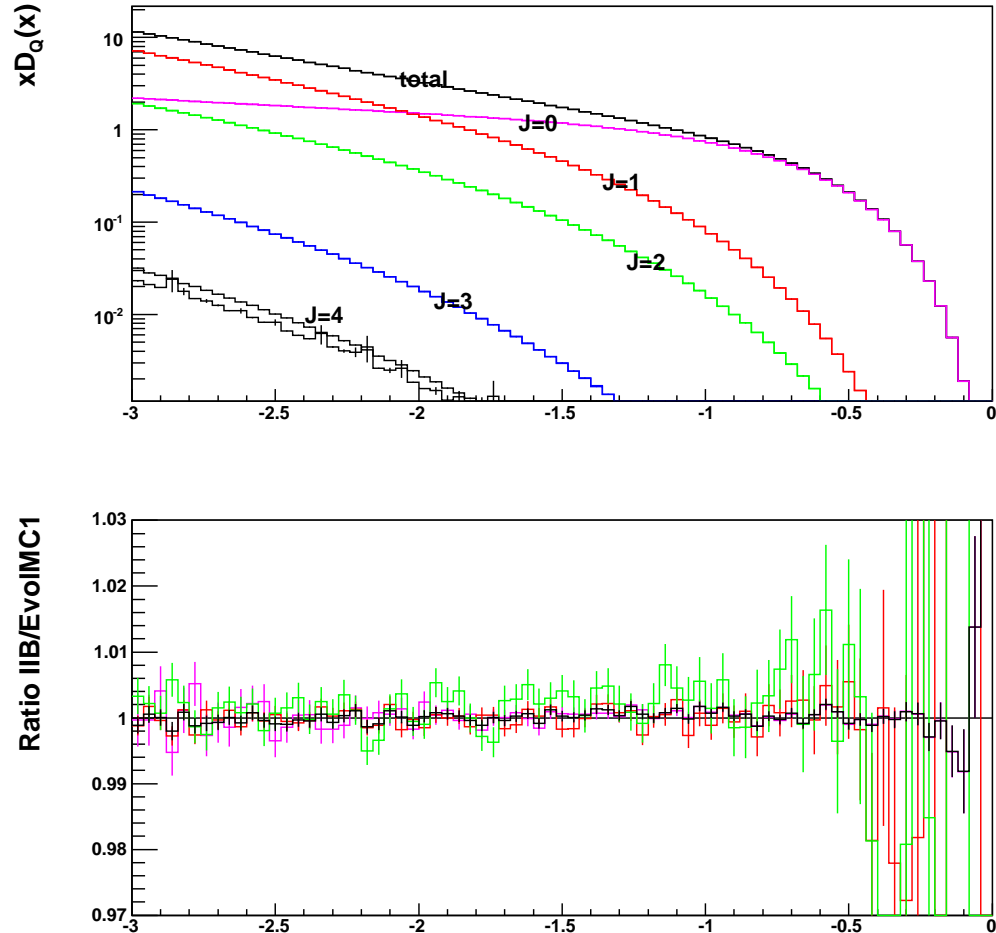
$n = 2: G \rightarrow Q \rightarrow G \rightarrow Q$, etc.

$n = 3: G \rightarrow Q \rightarrow G \rightarrow Q$, etc.

$n = 4: Q \rightarrow G \rightarrow Q \rightarrow G \rightarrow Q$, etc. "Total" is the sum of $n = 0, 1, 2, 3, 4$.

Evolution from proton at $Q = 1\text{GeV}$ up to 1TeV . New non-Markovian CMC (EvoIMC) agrees with unconstrained Markovian MC (EvoIFMC) to within $\sim 0.2\%$!

Test of non-Markovian Constrained MC; HERWIG



$J = 0$: $Q \rightarrow Q$

$J = 1$: $G \rightarrow Q$ and any no. of gluon emissions out of Q and G ,

$J = 2$: $G \rightarrow Q \rightarrow G \rightarrow Q$, etc.

$J = 3$: $G \rightarrow Q \rightarrow G \rightarrow Q$, etc.

$J = 4$: $Q \rightarrow G \rightarrow Q \rightarrow G \rightarrow Q$, etc. "Total" is the sum of $n = 0, 1, 2, 3, 4$.

Evolution from proton at $Q = 1\text{GeV}$ up to 1TeV . New non-Markovian CMC agrees with Markovian MC (MMC) to within $\sim 0.2\%$!

NEW (June 2005) and UNPUBLISHED!!!

Summary and outlook

- It is demonstrated using prototype program that the Constrained MC works in practice for the HERWIG evolution and for standard LL DGLAP with Quark–Gluon transitions.
- Still to be done soon: Including the rest of NLL corrections into CMC, mapping into full $d = 4$ phase space, and more...
- How to exploit this new technology in the construction of the full scale parton shower MC? To be seen...
- Most likely application: unified approach with unintegrated PDFs (CCFM style) and parton shower MC, up to NLL.

Most of presented new results: works with M. Skrzypek.